Lawrence Livermore National Laboratory

HYPRE: High Performance Preconditioners

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The *hypre* Team

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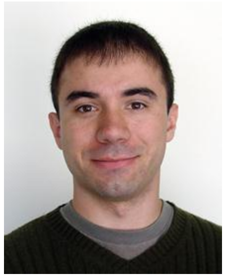

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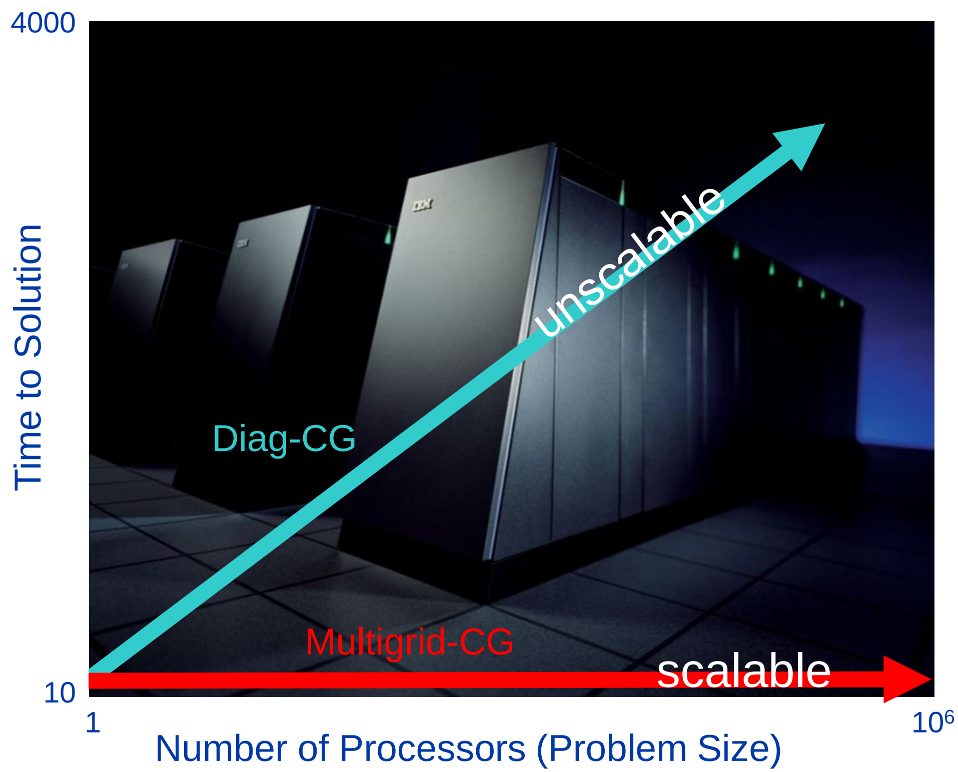
Outline

- Introduction / Motivation
- Getting Started / Linear System Interfaces

- Structured-Grid Interface (`Struct`)
- Semi-Structured-Grid Interface (`SStruct`)
- Finite Element Interface (`FEI`)
- Linear-Algebraic Interface (`IJ`)

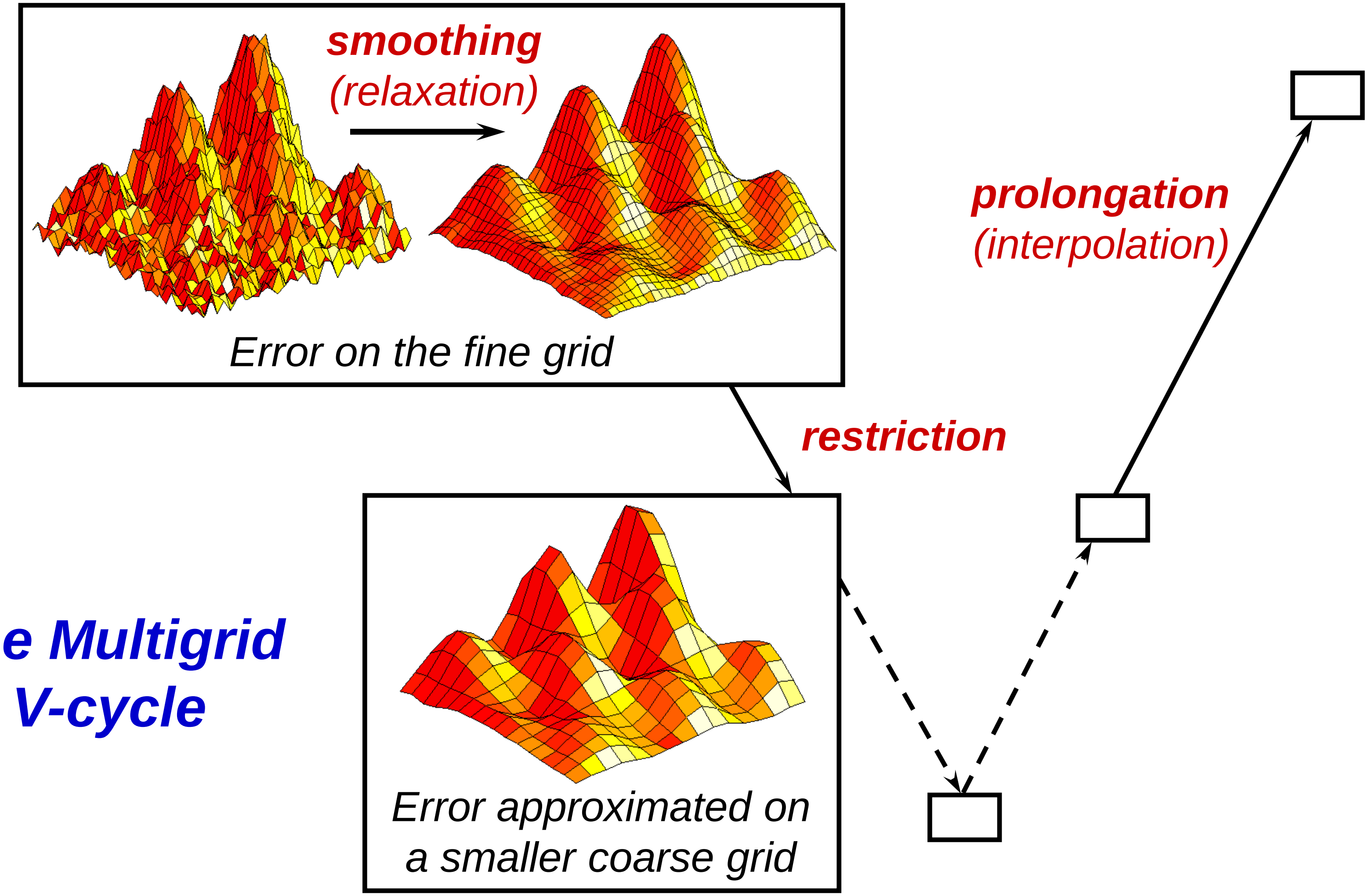
- Solvers and Preconditioners
- Additional Information

Multigrid linear solvers are optimal ($O(N)$ operations), and hence have good scaling potential



- Weak scaling – want constant solution time as problem size grows in proportion to the number of processors

Multigrid uses a sequence of coarse grids to accelerate the fine grid solution



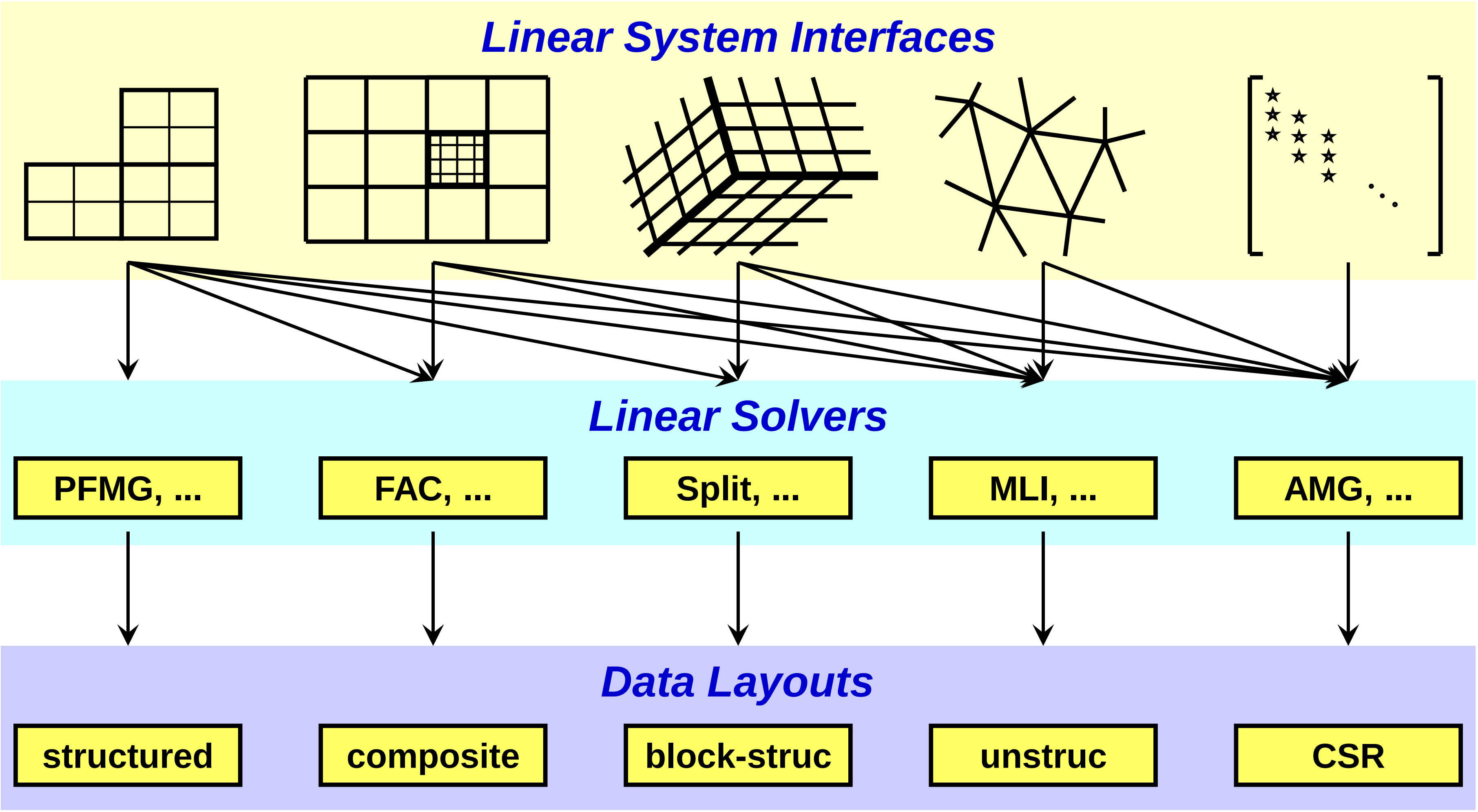
The Multigrid V-cycle

Getting Started

- Before writing your code:
  - choose a linear system interface
  - choose a solver / preconditioner
  - choose a matrix type that is compatible with your solver / preconditioner and system interface

- Now write your code:
  - build auxiliary structures (e.g., grids, stencils)
  - build matrix/vector through system interface
  - build solver/preconditioner
  - solve the system
  - get desired information from the solver

(Conceptual) linear system interfaces are necessary to provide “best” solvers and data layouts

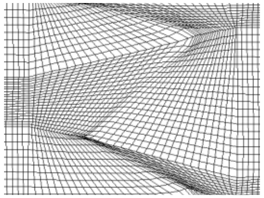


Why multiple interfaces? The key points

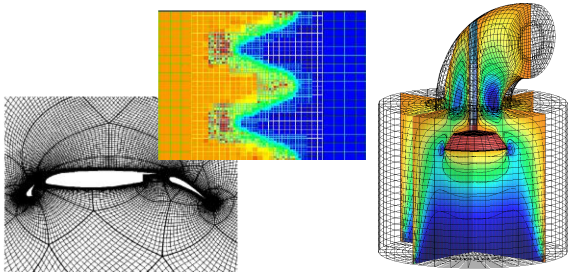
- Provides natural “views” of the linear system
- Eases some of the coding burden for users by eliminating the need to map to rows/columns
- Provides for more efficient (scalable) linear solvers
- Provides for more effective data storage schemes and more efficient computational kernels

Currently, *hypre* supports four system interfaces

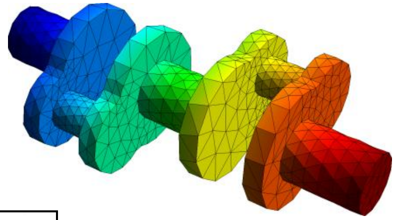
- Structured-Grid (`Struct`)
  - *logically rectangular grids*



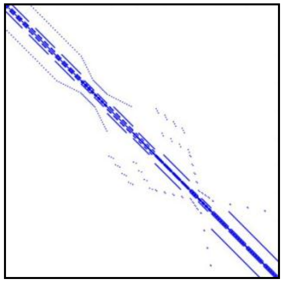
- Semi-Structured-Grid (`SStruct`)
  - *grids that are mostly structured*



- Finite Element (`FEI`)
  - *unstructured grids with finite elements*



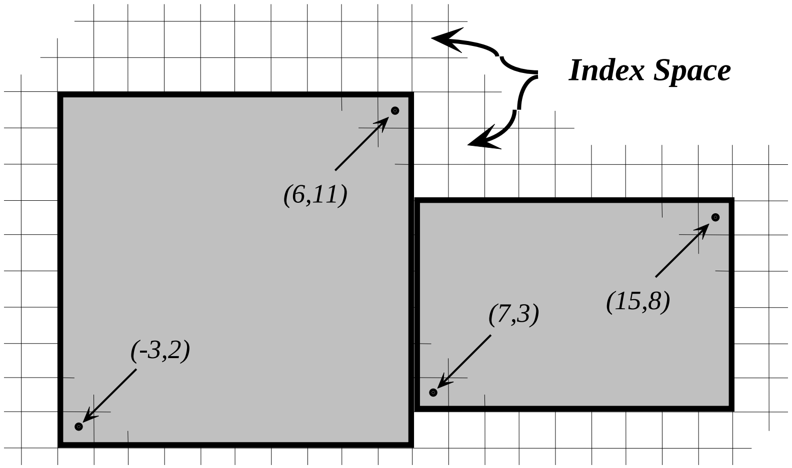
- Linear-Algebraic (`IJ`)
  - *general sparse linear systems*



- More about the first two next…

Structured-Grid System Interface (`Struct`)

- Appropriate for scalar applications on structured grids with a fixed stencil pattern
- Grids are described via a global *d*-dimensional *index space* (singles in 1D, tuples in 2D, and triples in 3D)
- A *box* is a collection of cell-centered indices, described by its “lower” and “upper” corners
- The scalar grid data is always associated with cell centers (unlike the more general SStruct interface)



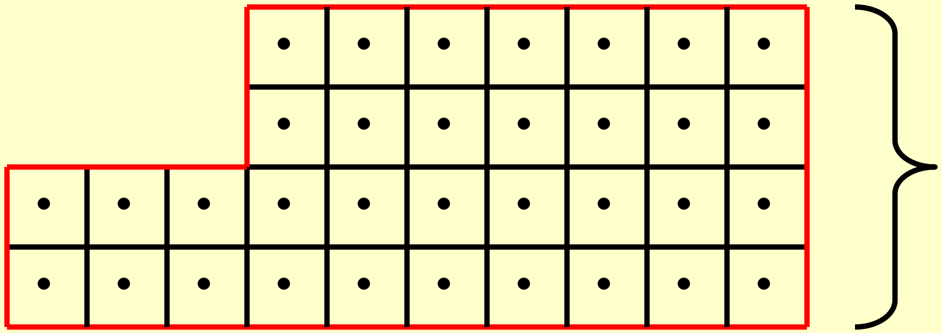
Structured-Grid System Interface (`Struct`)

- There are four basic steps involved:
  - set up the `Grid`
  - set up the `Stencil`
  - set up the `Matrix`
  - set up the right-hand-side `Vector`

- Consider the following 2D Laplacian problem

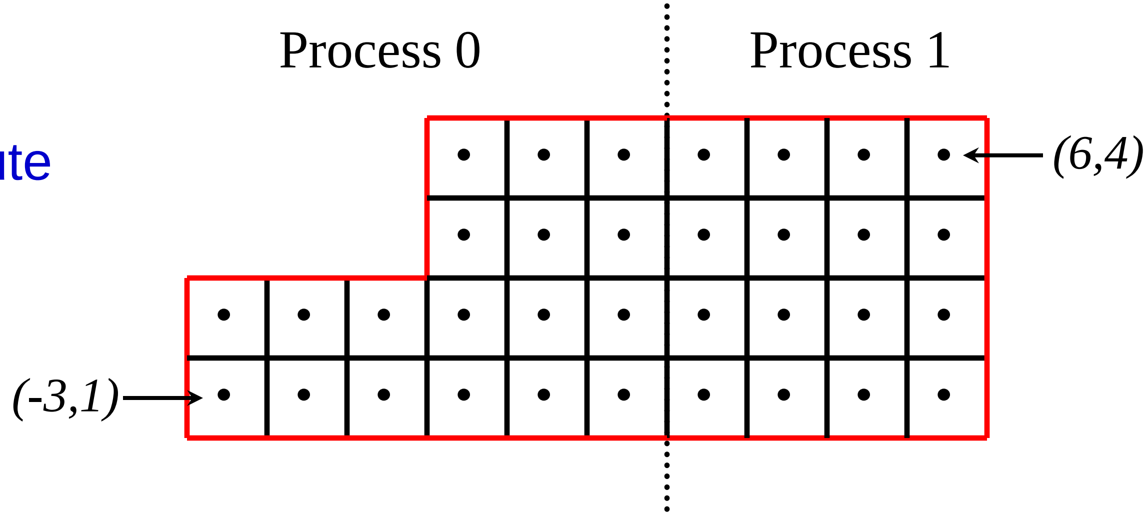
$$\begin{cases} -\nabla^2 u = f & \text{in the domain} \\ u = g & \text{on the boundary} \end{cases}$$

Structured-grid finite volume example:

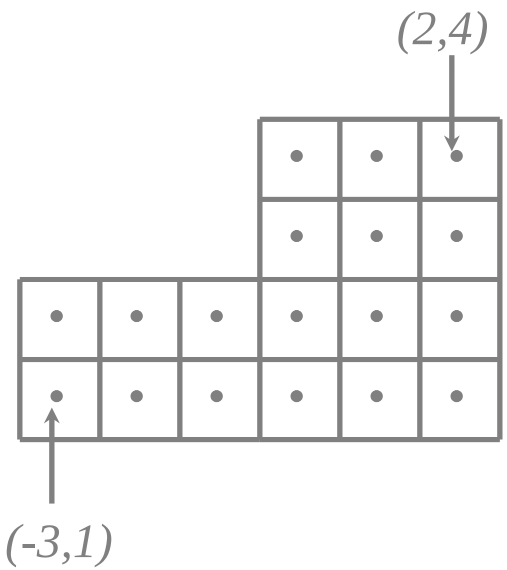


Standard 5-point finite volume discretization

Partition and distribute



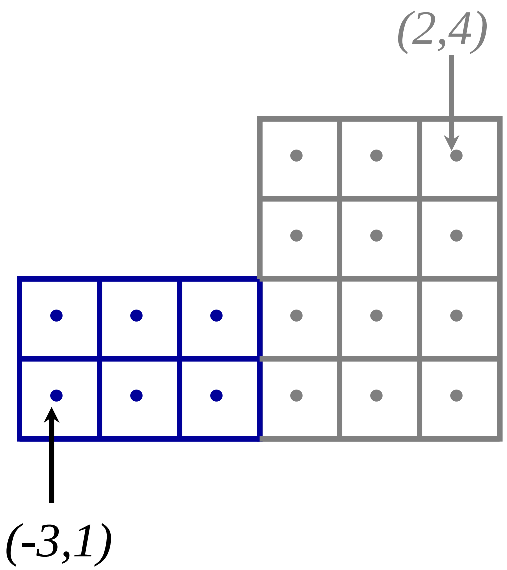
Structured-grid finite volume example:
Setting up the grid on process 0



Create the grid object

```
HYPRE_StructGrid grid;
int ndim   = 2;

HYPRE_StructGridCreate(MPI_COMM_WORLD, ndim, &grid);
```

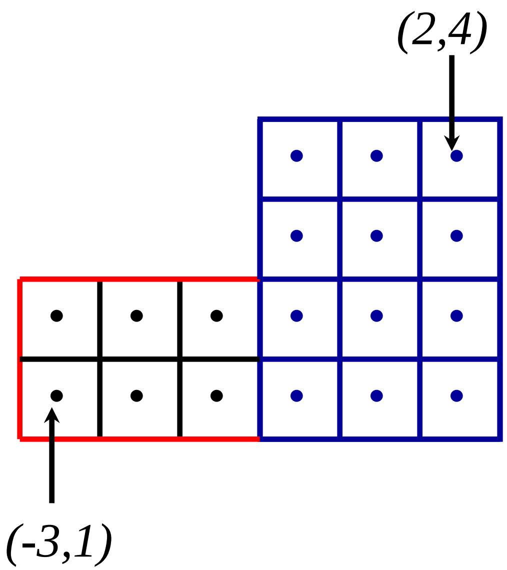
Structured-grid finite volume example:
Setting up the grid on process 0



Set grid extents for first box

```
int ilo0[2] = {-3,1};
int iup0[2] = {-1,2};

HYPRE_StructGridSetExtents(grid, ilo0, iup0);
```

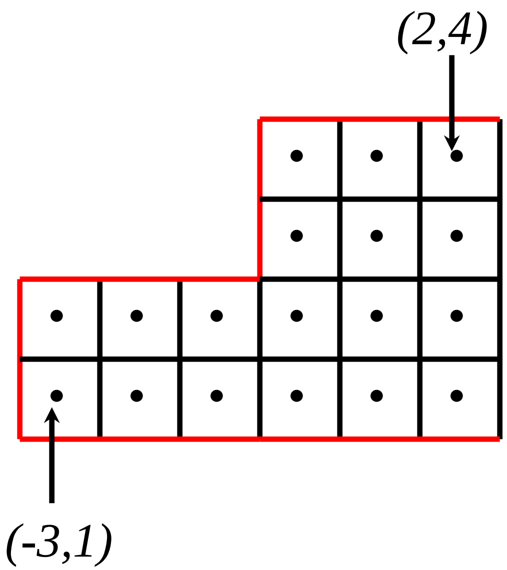
Structured-grid finite volume example:
Setting up the grid on process 0



Set grid extents for second box

```
int ilo1[2] = {0,1};
int iup1[2] = {2,4};

HYPRE_StructGridSetExtents(grid, ilo1, iup1);
```

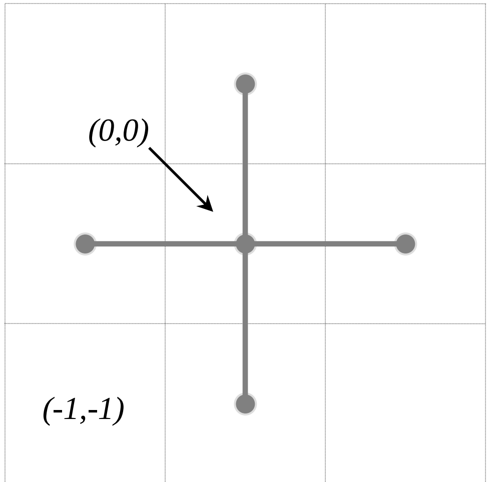
Structured-grid finite volume example:
Setting up the grid on process 0



Assemble the grid

```
HYPRE_StructGridAssemble(grid);
```

Structured-grid finite volume example:
Setting up the stencil (all processes)



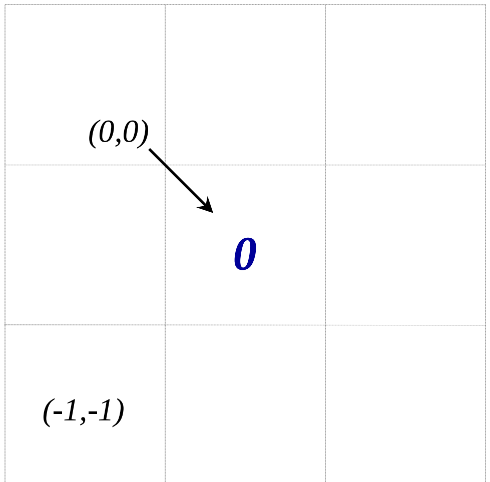
stencil entries		geometries
0	⟷	(0, 0)
1	⟷	(-1, 0)
2	⟷	(1, 0)
3	⟷	(0,-1)
4	⟷	(0, 1)

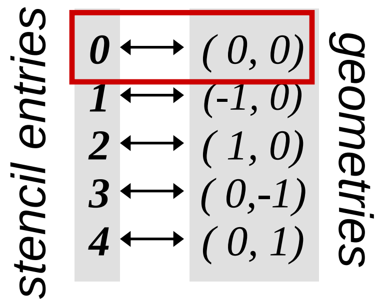
Create the stencil object

```
HYPRE_StructStencil stencil;
int ndim = 2;
int size = 5;

HYPRE_StructStencilCreate(ndim, size, &stencil);
```

Structured-grid finite volume example:

Setting up the stencil (all processes)

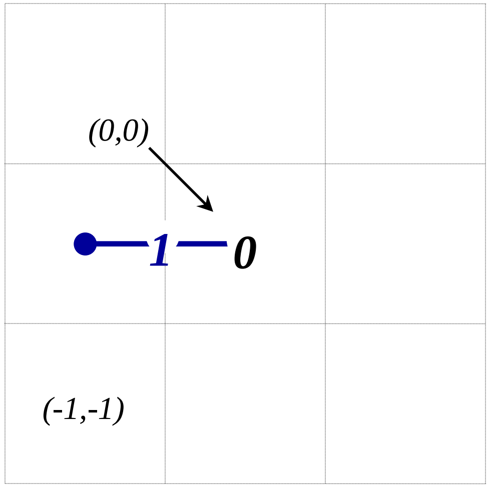


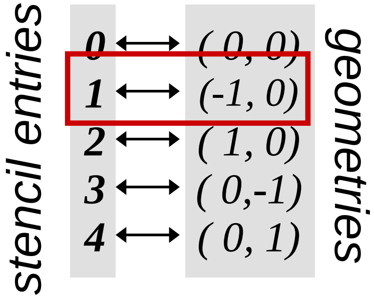


Set stencil entries

```
int entry = 0;
int offset[2] = {0,0};

HYPRE_StructStencilSetElement(stencil, entry, offset);
```

Structured-grid finite volume example:
Setting up the stencil (all processes)

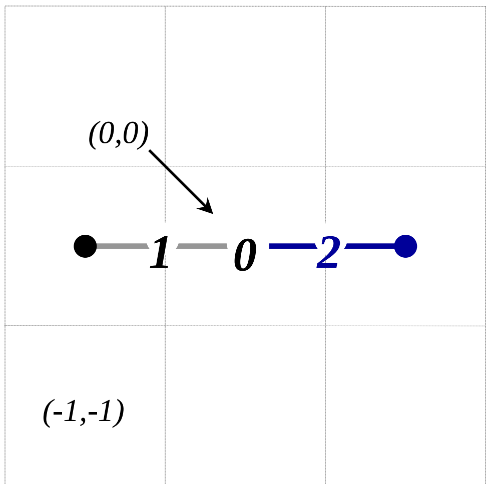


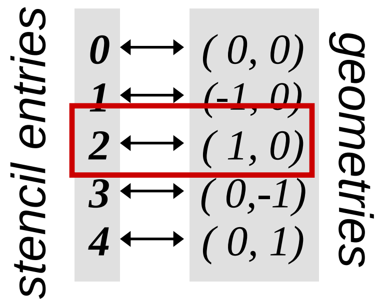


Set stencil entries

```
int entry = 1;
int offset[2] = {-1,0};

HYPRE_StructStencilSetElement(stencil, entry, offset);
```

Structured-grid finite volume example:
Setting up the stencil (all processes)

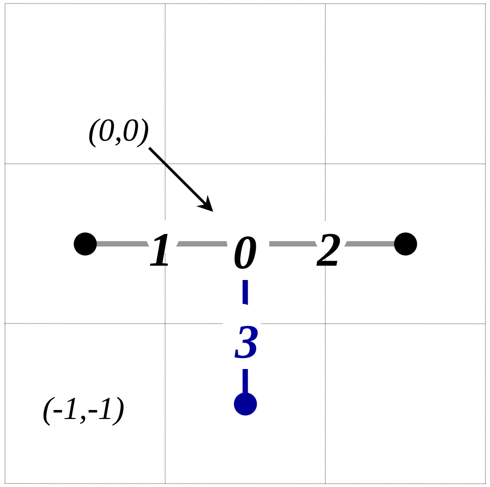


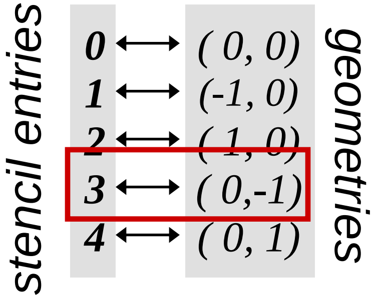


Set stencil entries

```
int entry = 2;
int offset[2] = {1,0};

HYPRE_StructStencilSetElement(stencil, entry, offset);
```

Structured-grid finite volume example:
Setting up the stencil (all processes)

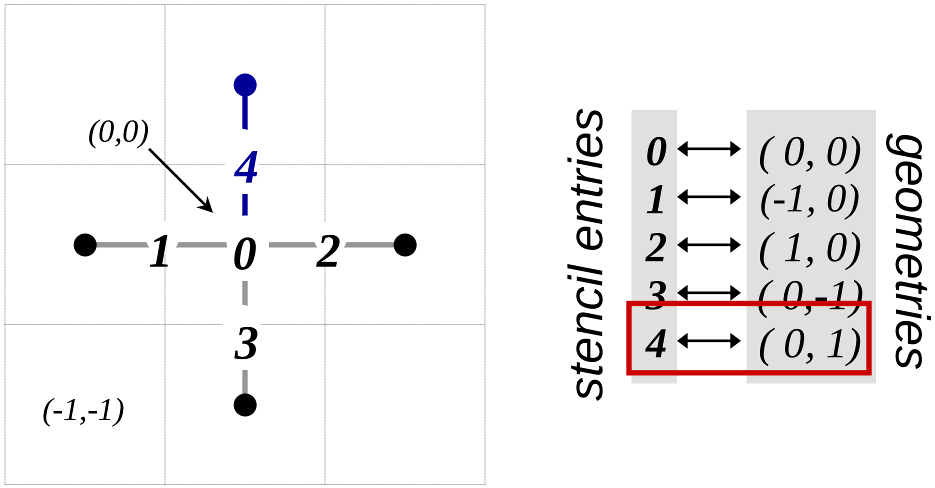




Set stencil entries

```
int entry = 3;
int offset[2] = {0,-1};

HYPRE_StructStencilSetElement(stencil, entry, offset);
```

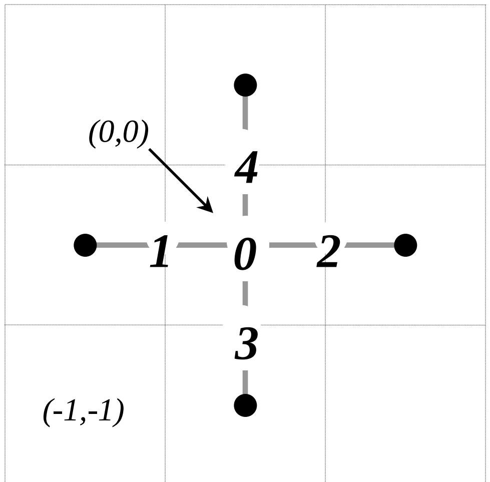
Structured-grid finite volume example:
Setting up the stencil (all processes)



Set stencil entries

```
int entry = 4;
int offset[2] = {0,1};

HYPRE_StructStencilSetElement(stencil, entry, offset);
```

Structured-grid finite volume example:
Setting up the stencil (all processes)

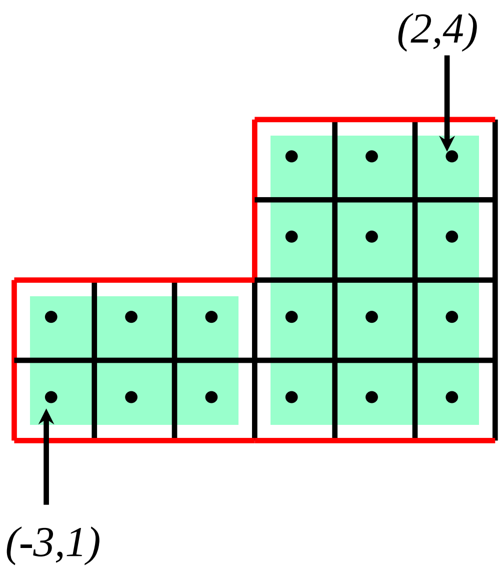


stencil entries		geometries
0	⟷	*(0, 0)*
1	⟷	*(-1, 0)*
2	⟷	*(1, 0)*
3	⟷	*(0,-1)*
4	⟷	*(0, 1)*

That's it!

There is no assemble routine

Structured-grid finite volume example :
Setting up the matrix on process 0

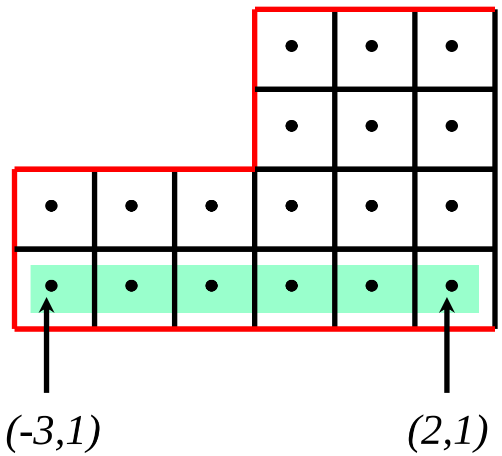


$$\begin{bmatrix} & S4 & \\ S1 & S0 & S2 \\ & S3 & \end{bmatrix} = \begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{bmatrix}$$

```
HYPRE_StructMatrix  A;
double  vals[24] = {4, -1, 4, -1, …};
int  nentries = 2;
int  entries[2] = {0,3};

HYPRE_StructMatrixCreate(MPI_COMM_WORLD,
   grid, stencil, &A);
HYPRE_StructMatrixInitialize(A);

HYPRE_StructMatrixSetBoxValues(A,
   ilo0, iup0, nentries, entries, vals);
HYPRE_StructMatrixSetBoxValues(A,
   ilo1, iup1, nentries, entries, vals);

/* set boundary conditions */
…
HYPRE_StructMatrixAssemble(A);
```

Structured-grid finite volume example :

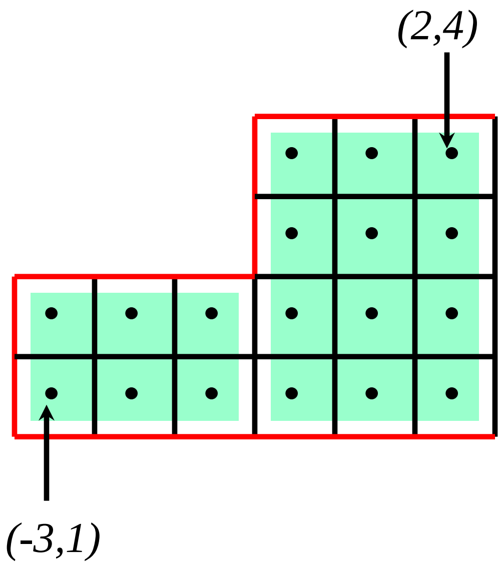
Setting up the matrix bc's on process 0



$$\begin{bmatrix} & S4 & \\ S1 & S0 & S2 \\ & S3 & \end{bmatrix} = \begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & 0 & \end{bmatrix}$$

```
int  ilo[2] = {-3, 1};
int  iup[2] = { 2, 1};
double  vals[6] = {0, 0, …};
int nentries = 1;

/* set interior coefficients */
…

/* implement boundary conditions */
…

i = 3;
HYPRE_StructMatrixSetBoxValues(A,
   ilo, iup, nentries, &i, vals);

/* complete implementation of bc's */
…
```

A structured-grid finite volume example :
Setting up the right-hand-side vector on process 0




```
HYPRE_StructVector  b;
double  vals[12] = {0, 0, …};

HYPRE_StructVectorCreate(MPI_COMM_WORLD,
   grid, &b);
HYPRE_StructVectorInitialize(b);

HYPRE_StructVectorSetBoxValues(b,
   ilo0, iup0, vals);
HYPRE_StructVectorSetBoxValues(b,
   ilo1, iup1, vals);

HYPRE_StructVectorAssemble(b);
```

Symmetric Matrices

- Some solvers support symmetric storage

- Between `Create()` and `Initialize`(), call:

  ```
  HYPRE_StructMatrixSetSymmetric(A, 1);
  ```

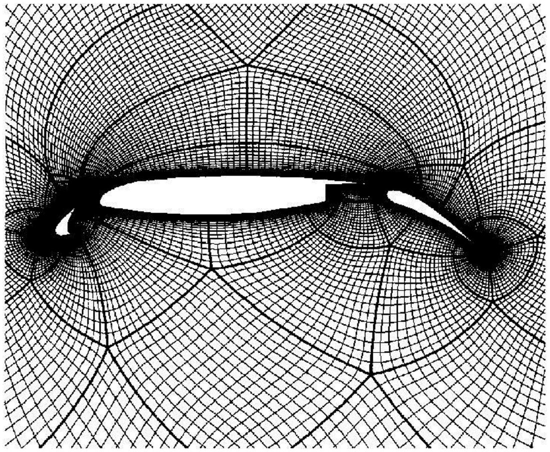
- For best efficiency, only set half of the coefficients

$$\begin{bmatrix} & (0,1) & \\ & (0,0) & (1,0) \\ & & \end{bmatrix} \Leftrightarrow \begin{bmatrix} & \mathrm{S2} & \\ & \mathrm{S0} & \mathrm{S1} \\ & & \end{bmatrix}$$

- This is enough info to recover the full 5-pt stencil

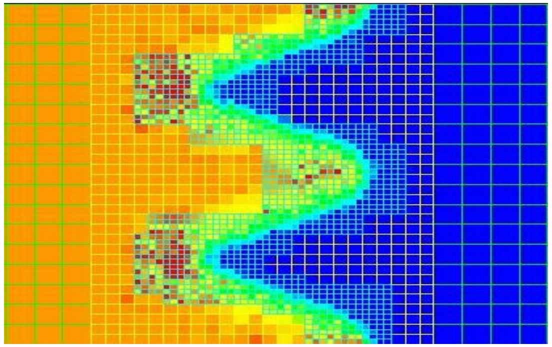
Semi-Structured-Grid System Interface (`SStruct`)

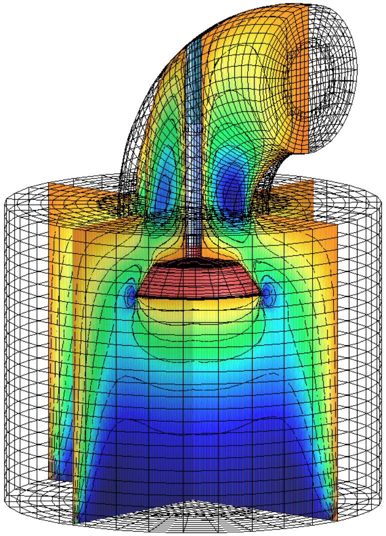
- Allows more general grids:
  - Grids that are mostly (but not entirely) structured
  - Examples: *block-structured grids, structured adaptive mesh refinement grids, overset grids*



Block-Structured

Adaptive Mesh Refinement

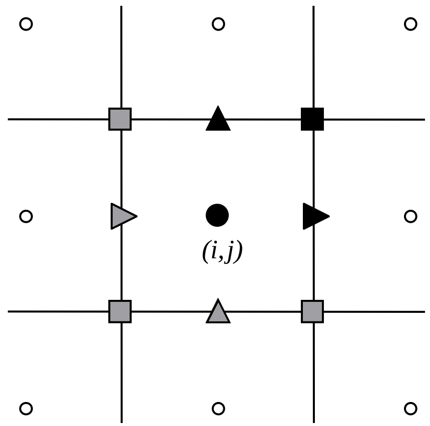




Overset

Semi-Structured-Grid System Interface (`SStruct`)

- Allows more general PDE's
  - Multiple variables (system PDE's)
  - Multiple variable types (cell centered, face centered, vertex centered, …)



Variables are referenced by the abstract cell-centered index to the left and down

Semi-Structured-Grid System Interface (`SStruct`)

- The `SStruct` grid is composed out of structured grid *parts*
- The interface uses a *graph* to allow nearly arbitrary relationships between part data
- The graph is constructed from stencils or finite element stiffness matrices (new) plus additional data-coupling information set either
  - directly with `GraphAddEntries()`, or
  - by relating parts with `GridSetNeighborPart()` and `GridSetSharedPart()` (new)
- We will consider three examples:
  - block-structured grid using stencils
  - star-shaped grid with finite elements (new)
  - structured adaptive mesh refinement

Semi-Structured-Grid System Interface (`SStruct`)

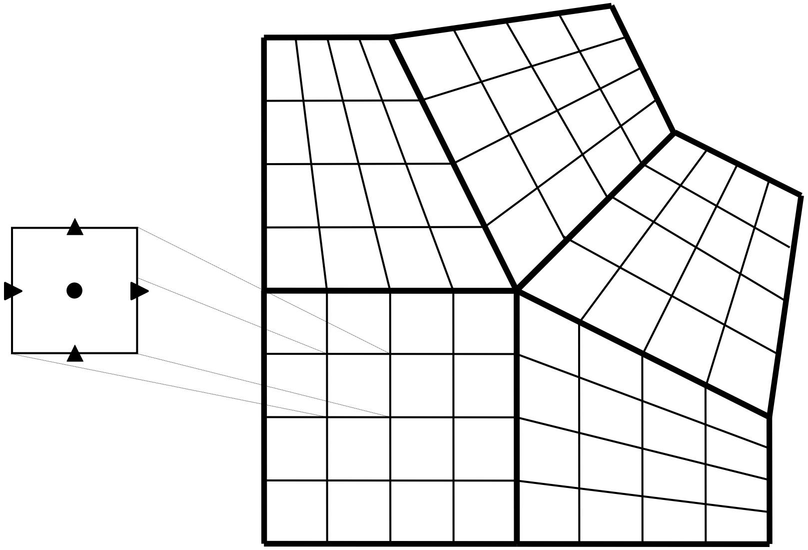
- There are five basic steps involved:
  - set up the `Grid`
  - set up the `Stencils`
  - set up the `Graph`
  - set up the `Matrix`
  - set up the right-hand-side `Vector`

Block-structured grid example (`SStruct`)

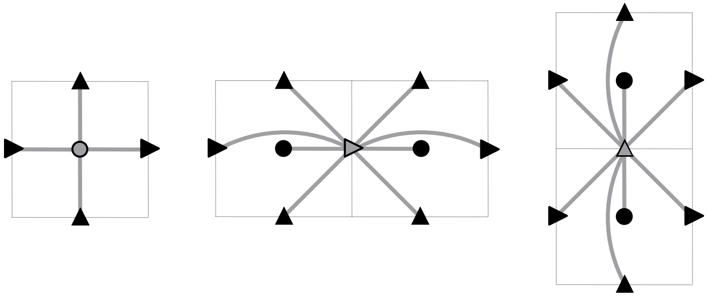
- Consider the following block-structured grid discretization of the diffusion equation

$$-\nabla \cdot \mathbf{K} \nabla u + \sigma u = f$$

A block-structured grid with 3 variable types



The 3 discretization stencils

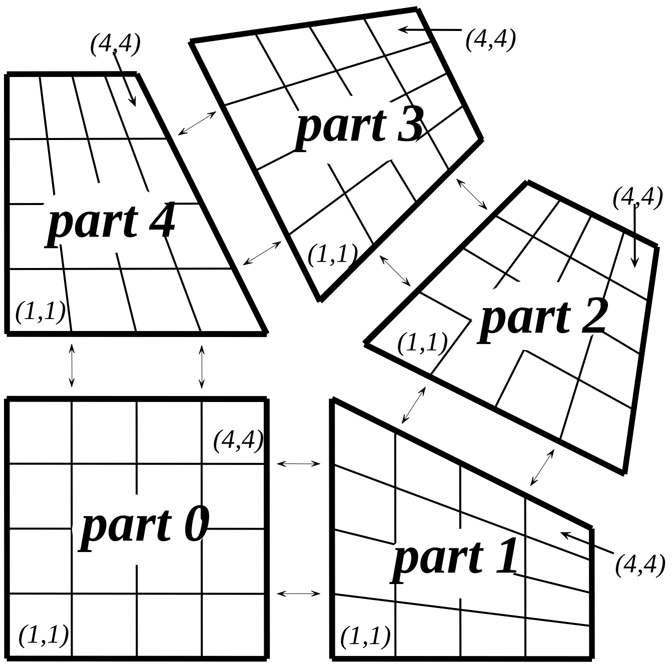


Block-structured grid example (`SStruct`)

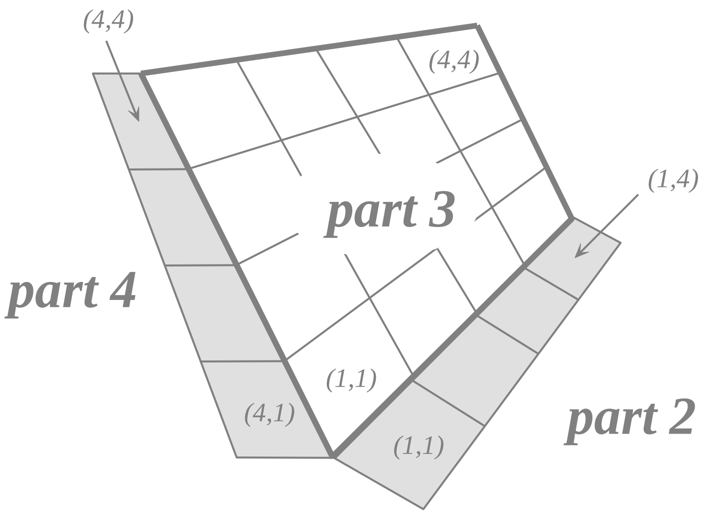
- The `Grid` is described via 5 logically-rectangular parts

- We assume 5 processes such that process p owns part p (user defines the distribution)

- We consider the interface calls made by process 3



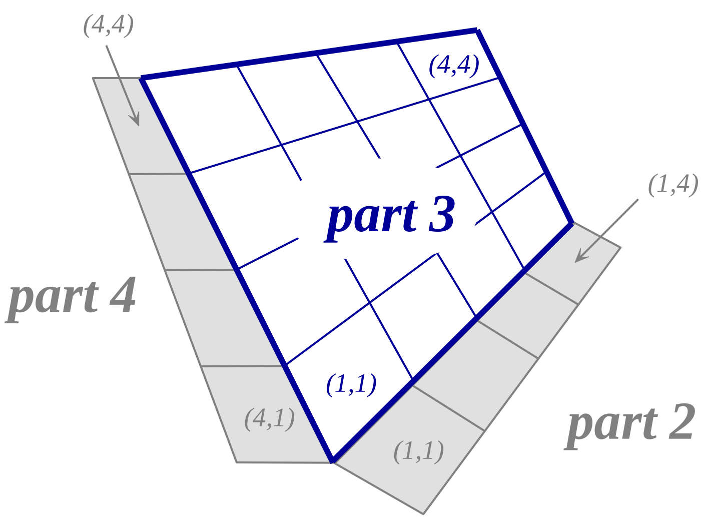
Block-structured grid example:
Setting up the grid on process 3



Create the grid object

```
HYPRE_SStructGrid grid;
int ndim   = 2;
int nparts = 5;

HYPRE_SStructGridCreate(MPI_COMM_WORLD, ndim, nparts, &grid);
```

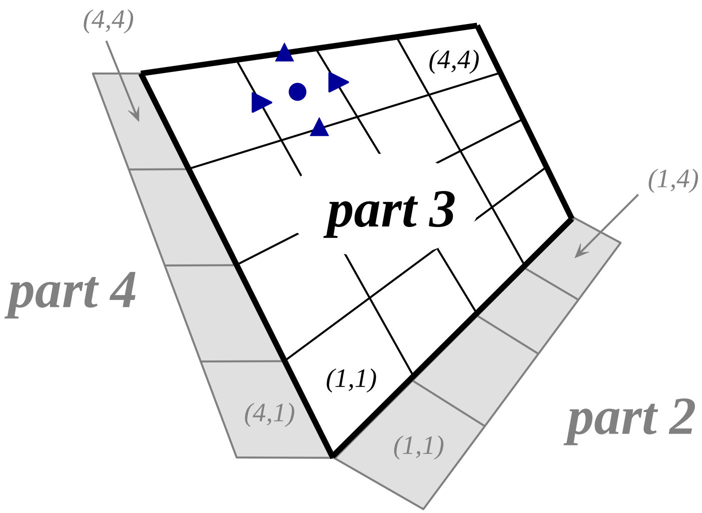
Block-structured grid example:
Setting up the grid on process 3



Set grid extents for part 3

```
int part = 3;
int ilower[2] = {1,1};
int iupper[2] = {4,4};

HYPRE_SStructGridSetExtents(grid, part, ilower, iupper);
```

Block-structured grid example:
Setting up the grid on process 3

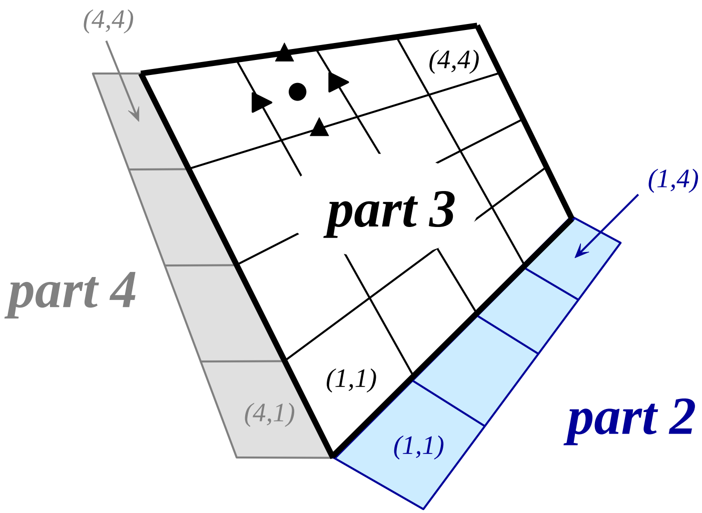


Set grid variables for each part

```
int part, nvars = 3;
int vartypes[3] = {HYPRE_SSTRUCT_VARIABLE_CELL,
                   HYPRE_SSTRUCT_VARIABLE_XFACE,
                   HYPRE_SSTRUCT_VARIABLE_YFACE};

HYPRE_SStructGridSetVariables(grid, part, nvars, vartypes);
```

Block-structured grid example:

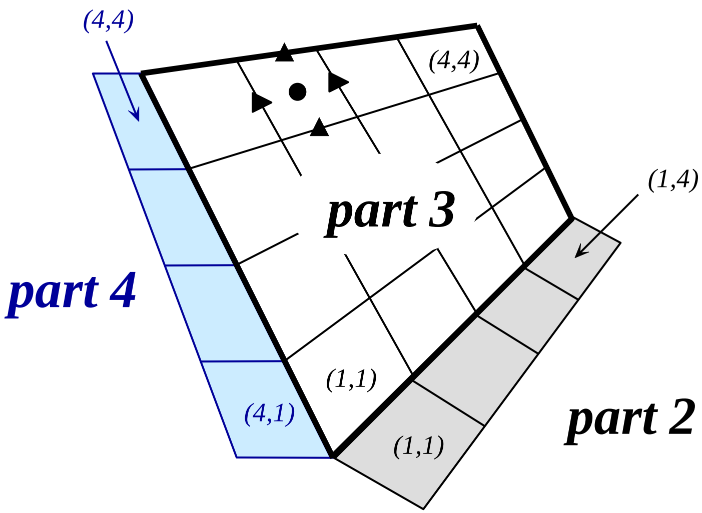
Setting up the grid on process 3



Set spatial relationship between parts 3 and 2

```
int part = 3, nbor_part = 2;
int ilower[2] = {1,0}, iupper[2] = {4,0};
int nbor_ilower[2] = {1,1}, nbor_iupper[2] = {1,4};
int index_map[2] = {1,0}, index_dir[2] = {1,-1};

HYPRE_SStructGridSetNeighborPart(grid, part, ilower, iupper,
nbor_part, nbor_ilower, nbor_iupper, index_map, index_dir);
```

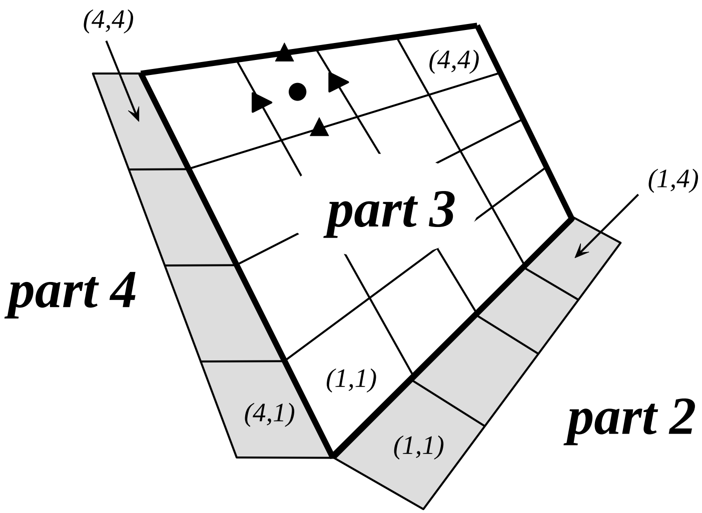
Block-structured grid example:
Setting up the grid on process 3



Set spatial relationship between parts 3 and 4

```
int part = 3, nbor_part = 4;
int ilower[2] = {0,1}, iupper[2] = {0,4};
int nbor_ilower[2] = {4,1}, nbor_iupper[2] = {4,4};
int index_map[2] = {0,1}, index_dir[2] = {1,1};

HYPRE_SStructGridSetNeighborPart(grid, part, ilower, iupper,
nbor_part, nbor_ilower, nbor_iupper, index_map, index_dir);
```

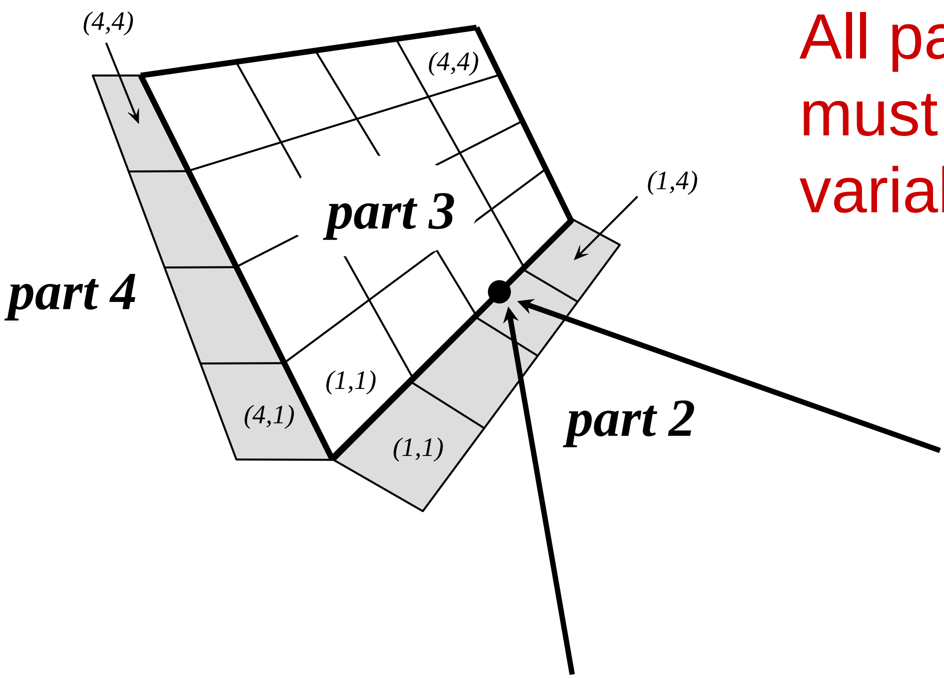
Block-structured grid example:
Setting up the grid on process 3



Assemble the grid

```
HYPRE_SStructGridAssemble(grid);
```

Block-structured grid example:
some comments on **`SetNeighborPart()`**

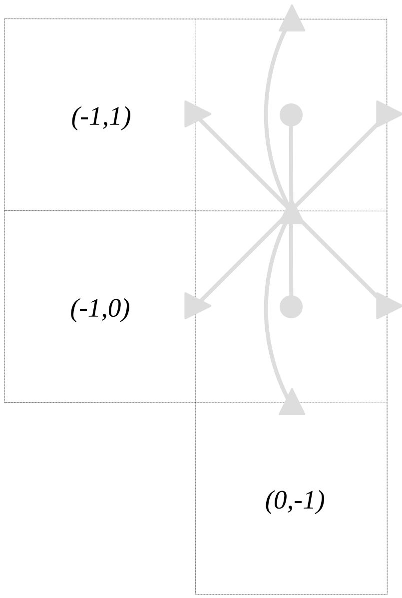


All parts related via this routine must have consistent lists of variables and types

Some variables on different parts become "the same"

Variables may have different types on different parts (e.g., *y-face* on part 3 and *x-face* on part 2)

Block-structured grid example:
Setting up the *y-face* stencil (all processes)



stencil entries / *geometries*

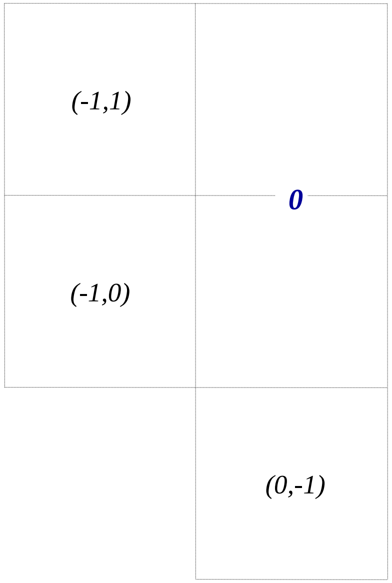
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

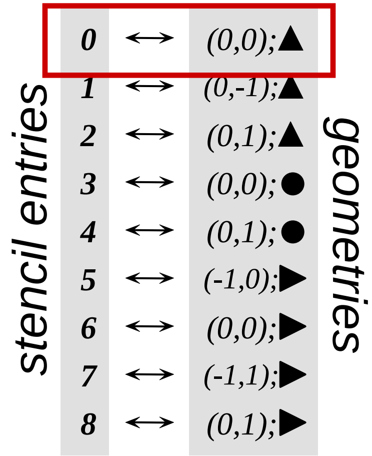
Create the stencil object

```
HYPRE_SStructStencil stencil;
int ndim = 2;
int size = 9;

HYPRE_SStructStencilCreate(ndim, size, &stencil);
```

Block-structured grid example:

Setting up the *y-face* stencil (all processes)

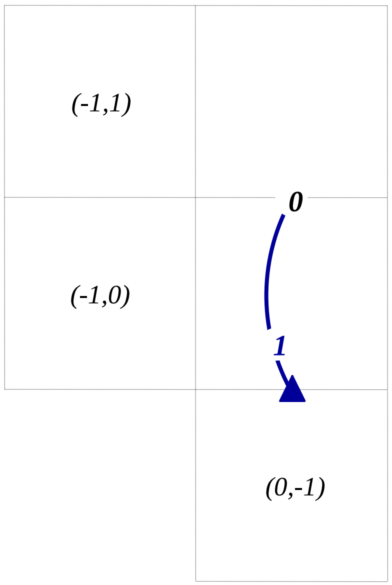




Set stencil entries

```
int entry = 0;
int offset[2] = {0,0};
int var = 2; /* the y-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)

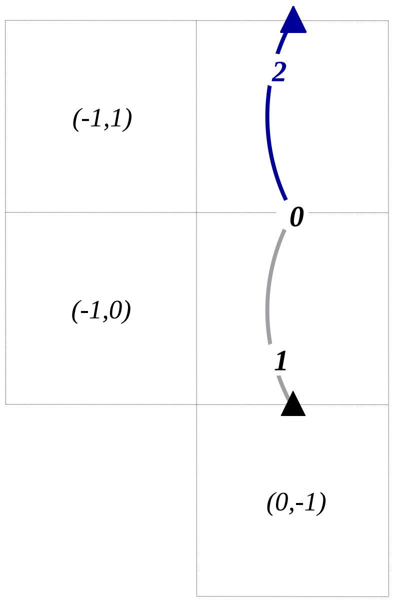


stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 1;
int offset[2] = {0,-1};
int var = 2; /* the y-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)

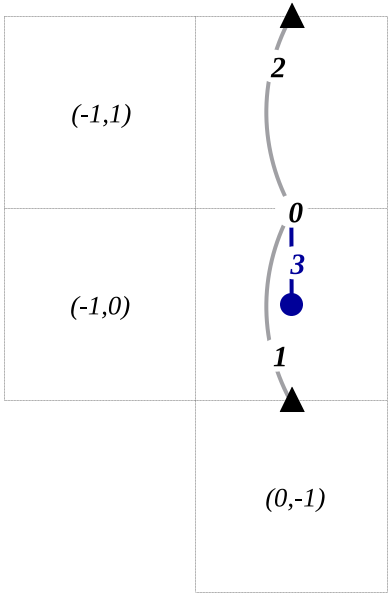


stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 2;
int offset[2] = {0,1};
int var = 2; /* the y-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)



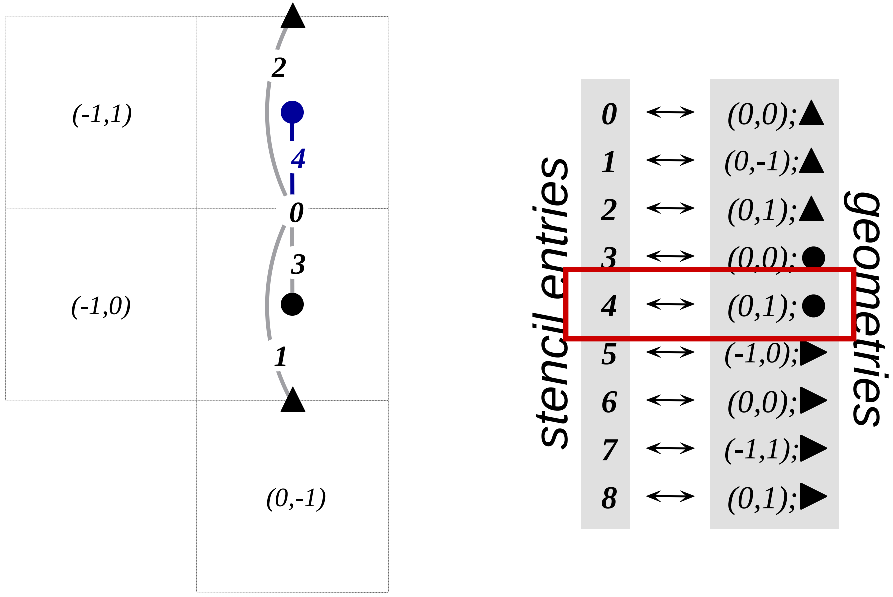
stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 3;
int offset[2] = {0,0};
int var = 0; /* the cell-centered variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:

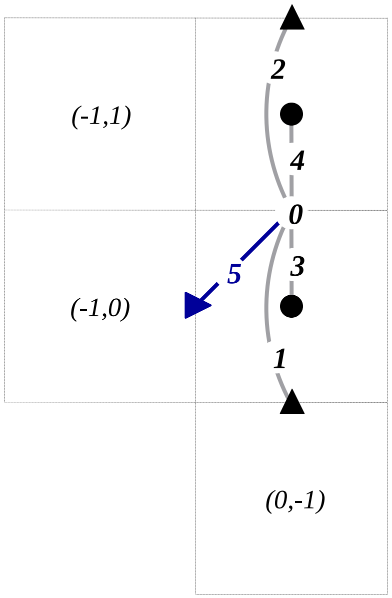
Setting up the *y-face* stencil (all processes)



Set stencil entries

```
int entry = 4;
int offset[2] = {0,1};
int var = 0; /* the cell-centered variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)



stencil entries ↔ geometries

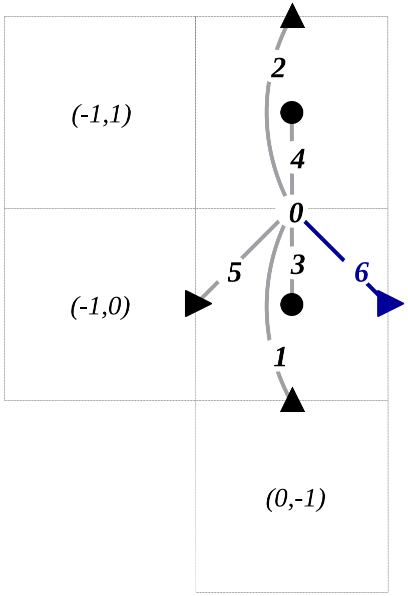
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 5;
int offset[2] = {-1,0};
int var = 1; /* the x-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:

Setting up the *y-face* stencil (all processes)

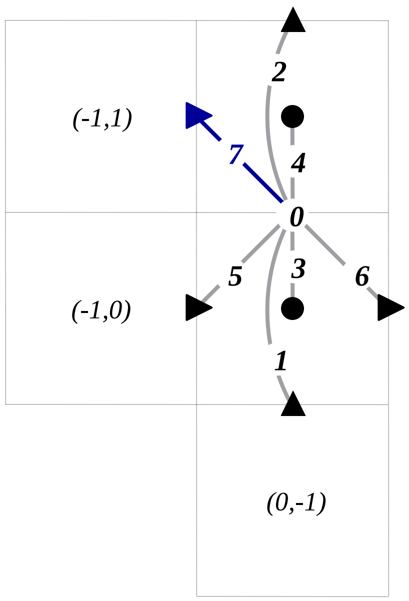


stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 6;
int offset[2] = {0,0};
int var = 1; /* the x-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)

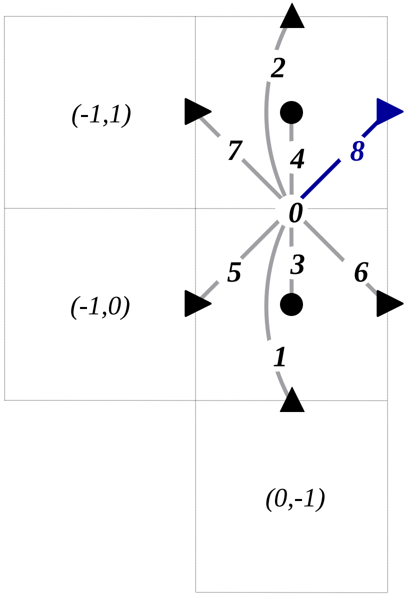


stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 7;
int offset[2] = {-1,1};
int var = 1; /* the x-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)



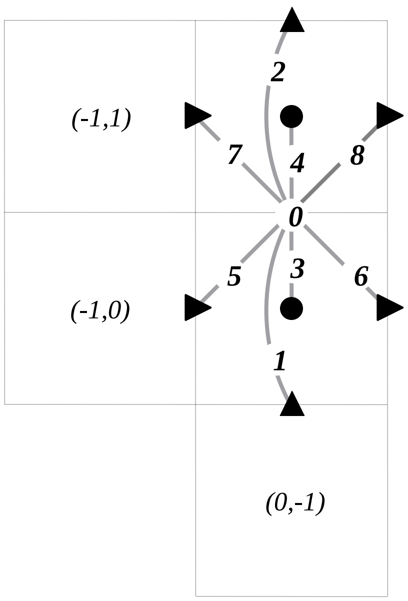
stencil entries ↔ geometries

stencil entries		geometries
0	↔	(0,0);▲
1	↔	(0,-1);▲
2	↔	(0,1);▲
3	↔	(0,0);●
4	↔	(0,1);●
5	↔	(-1,0);▶
6	↔	(0,0);▶
7	↔	(-1,1);▶
8	↔	(0,1);▶

Set stencil entries

```
int entry = 8;
int offset[2] = {0,1};
int var = 1; /* the x-face variable number */

HYPRE_SStructSetStencilEntry(stencil, entry, offset, var);
```

Block-structured grid example:
Setting up the *y-face* stencil (all processes)

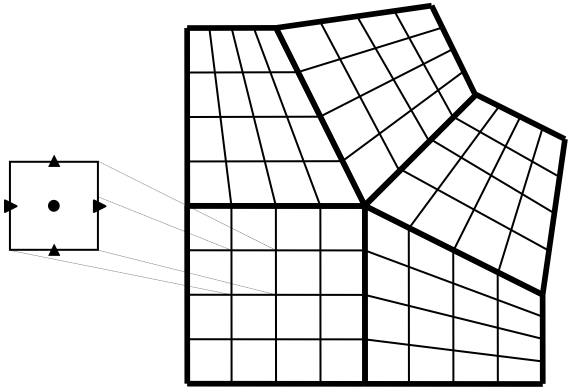


stencil entries		geometries
0	↔	*(0,0)*;▲
1	↔	*(0,-1)*;▲
2	↔	*(0,1)*;▲
3	↔	*(0,0)*;●
4	↔	*(0,1)*;●
5	↔	*(-1,0)*;▶
6	↔	*(0,0)*;▶
7	↔	*(-1,1)*;▶
8	↔	*(0,1)*;▶

That's it!

There is no assemble routine

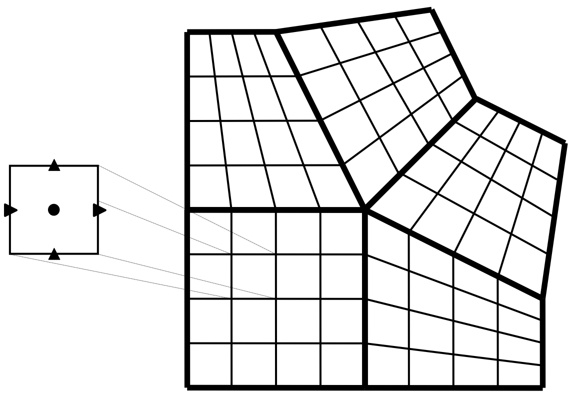
Block-structured grid example:
Setting up the graph on process 3

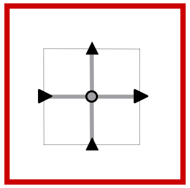


Create the graph object

```
HYPRE_SStructGraph graph;

HYPRE_SStructGraphCreate(MPI_COMM_WORLD, grid, &graph);
```

Block-structured grid example:
Setting up the graph on process 3

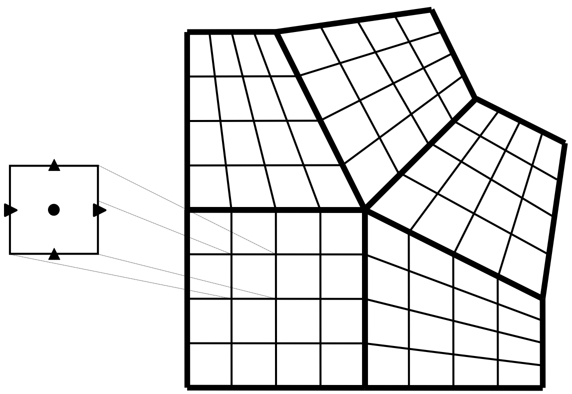


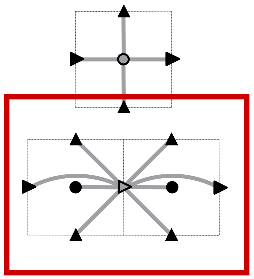


Set the cell-centered stencil for each part

```
int part;
int var = 0;
HYPRE_SStructStencil cell_stencil;

HYPRE_SStructGraphSetStencil(graph, part, var, cell_stencil);
```

Block-structured grid example:
Setting up the graph on process 3

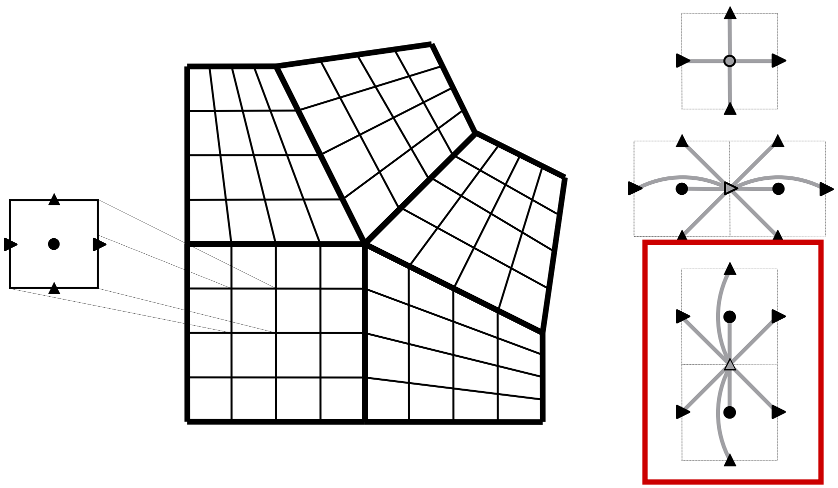




Set the x-face stencil for each part

```
int part;
int var = 1;
HYPRE_SStructStencil x_stencil;

HYPRE_SStructGraphSetStencil(graph, part, var, x_stencil);
```

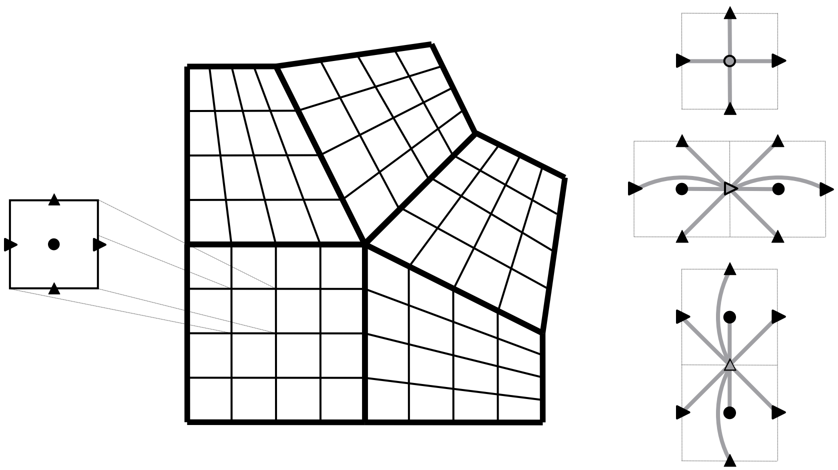
Block-structured grid example:
Setting up the graph on process 3



Set the y-face stencil for each part

```
int part;
int var = 2;
HYPRE_SStructStencil y_stencil;

HYPRE_SStructGraphSetStencil(graph, part, var, y_stencil);
```

Block-structured grid example:
Setting up the graph on process 3



Assemble the graph

```
/* No need to add non-stencil entries
 * with HYPRE_SStructGraphAddEntries() */

HYPRE_SStructGraphAssemble(graph);
```

Block-structured grid example:
Setting up the matrix and vector

- The matrix and vector objects are constructed in a manner similar to the `Struct` interface

- Matrix coefficients are set with the routines
  - `HYPRE_SStructMatrixSetValues()`
  - `HYPRE_SStructMatrixAddToValues()`

- Vector values are set with similar routines
  - `HYPRE_SStructVectorSetValues()`
  - `HYPRE_SStructVectorAddToValues()`

New finite element (FEM) style interface for SStruct as an alternative to stencils

- Beginning with *hypre* version 2.6.0b

- `GridSetSharedPart()` is similar to `SetNeighborPart`, but allows one to specify shared cells, faces, edges, or vertices
- `GridSetFEMOrdering()` sets the ordering of the unknowns in an element (always a cell)
- `GraphSetFEM()` indicates that an FEM approach will be used to set values instead of a stencil approach
- `GraphSetFEMSparsity()` sets the nonzero pattern for the stiffness matrix
- `MatrixAddFEMValues()` and `VectorAddFEMValues()`

- See examples: `ex13.c`, `ex14.c`, and `ex15.c`

Finite Element (FEM) example (`SStruct`)

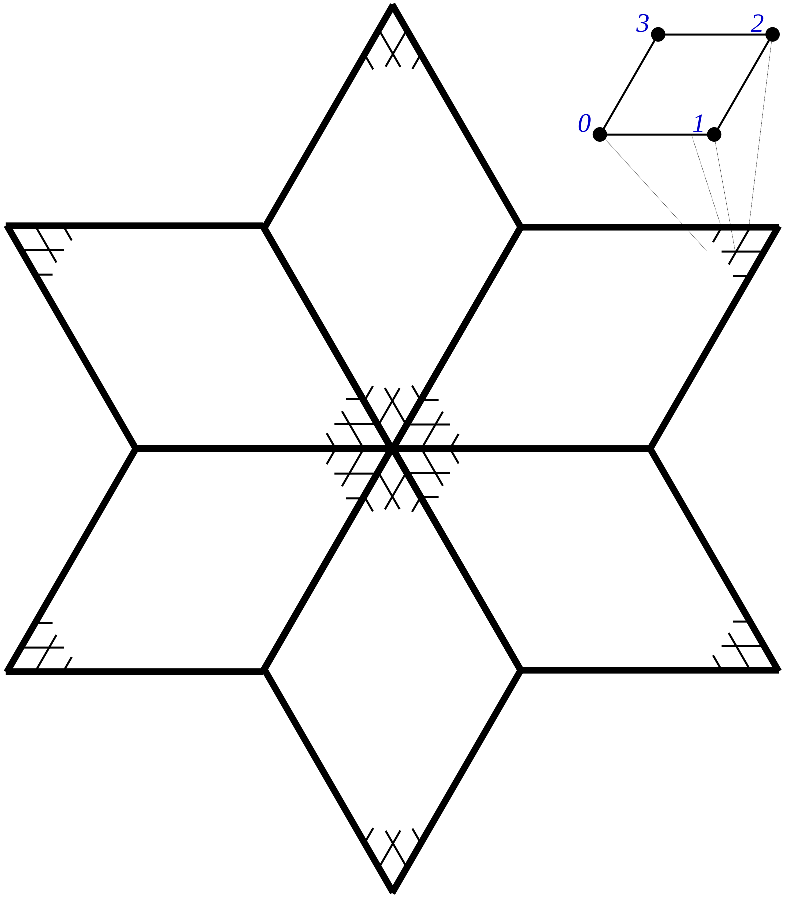
- FEM nodal discretization of the Laplace equation on a star-shaped domain

$$\begin{cases} -\nabla^2 u = 1 & \text{in } \Omega \\ u = 0 & \text{on } \Gamma \end{cases}$$

- FEM stiffness matrix

$$\begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 4-k & -1 & -2+k & -1 \\ -1 & 4+k & -1 & -2-k \\ -2+k & -1 & 4-k & -1 \\ -1 & -2-k & -1 & 4+k \end{pmatrix} \alpha \end{array}$$

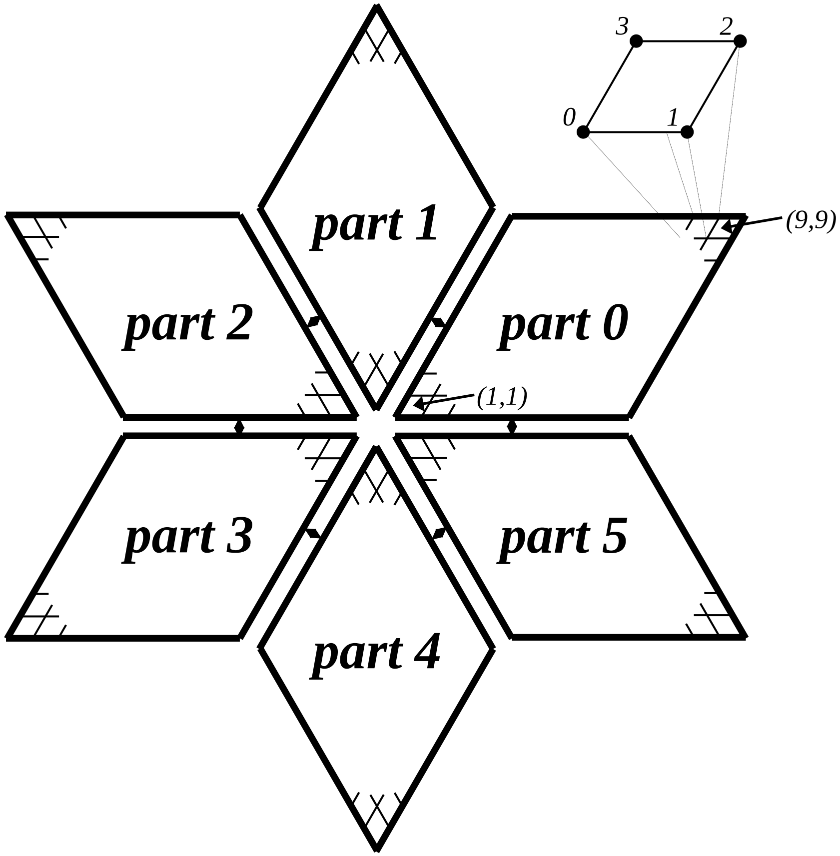
$$\alpha = (6\sin(\gamma))^{-1}, \quad k = 3\cos(\gamma), \quad \gamma = \pi/3$$



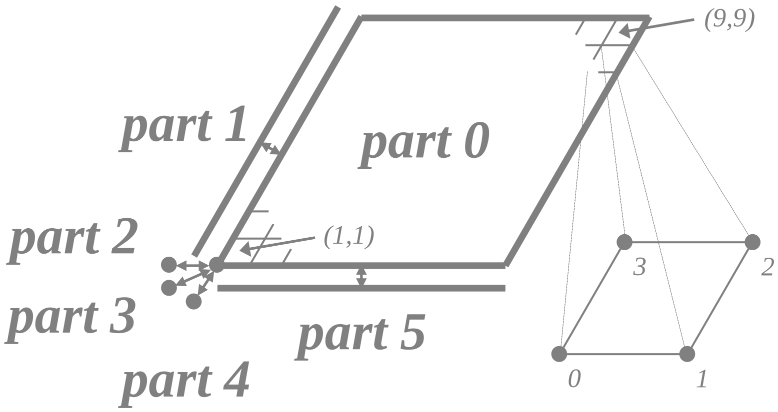
See example code `ex14.c`

FEM example (`SStruct`)

- The `Grid` is described via 6 logically-rectangular parts
- We assume 6 processes, where process p owns part p
- The `Matrix` is assembled from stiffness matrices (no stencils)
- We consider the interface calls made by process 0



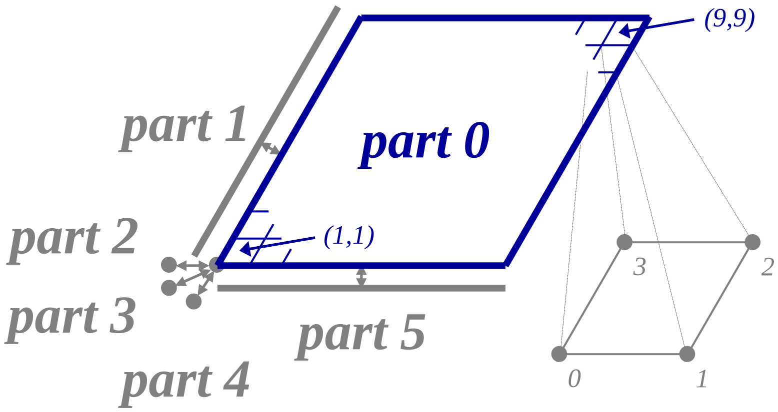
FEM example: Setting up the grid on process 0



Create the grid object

```
HYPRE_SStructGrid grid;
int ndim    = 2;
int nparts = 6;

HYPRE_SStructGridCreate(MPI_COMM_WORLD, ndim, nparts, &grid);
```

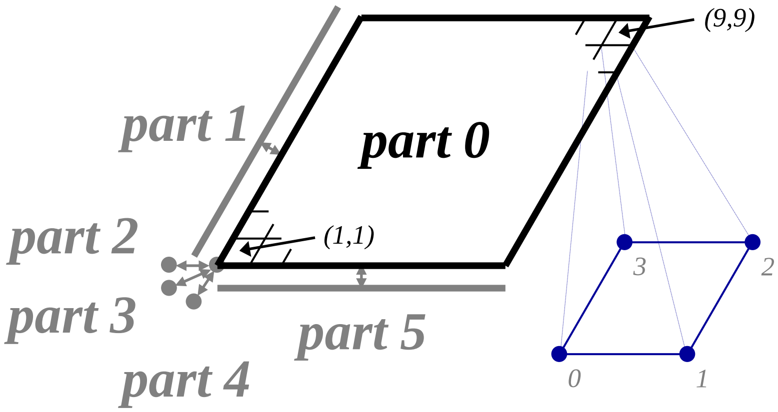
FEM example: Setting up the grid on process 0



Set grid extents for part 0

```
int part = 0;
int ilower[2] = {1,1};
int iupper[2] = {9,9};

HYPRE_SStructGridSetExtents(grid, part, ilower, iupper);
```

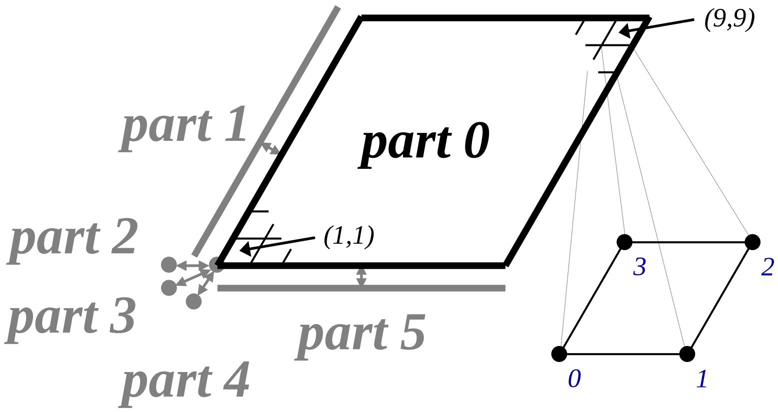
FEM example: Setting up the grid on process 0



Set grid variables for each part

```
int part;
int nvars = 1;
int vartypes[3] = {HYPRE_SSTRUCT_VARIABLE_NODE};

HYPRE_SStructGridSetVariables(grid, part, nvars, vartypes);
```

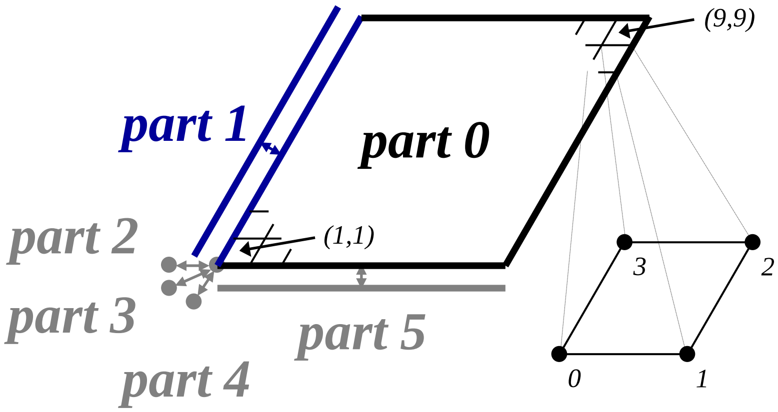
FEM example: Setting up the grid on process 0



Set FEM ordering of variables on part 0

```
int part = 0;
int ordering[12] = { 0, -1, -1,
                     0, +1, -1,
                     0, +1, +1,
                     0, -1, +1 };

HYPRE_SStructGridSetFEMOrdering(grid, part, ordering);
```

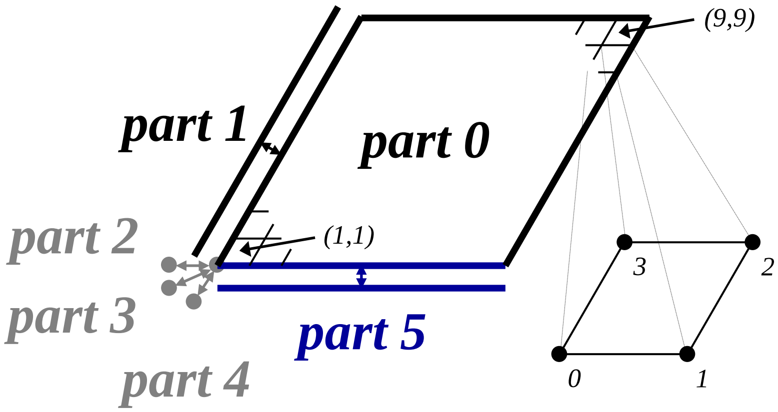
FEM example: Setting up the grid on process 0



Set shared variables for parts 0 and 1

```
int part = 0, spart = 1;
int ilo[2]  = {1,1}, iup[2]  = {1,9}, offset[2]  = {-1,0};
int silo[2] = {1,1}, siup[2] = {9,1}, soffset[2] = {0,-1};
int index_map[2] = {1,0}, index_dir[2] = {-1,1};

HYPRE_SStructGridSetSharedPart(grid, part, ilo, iup, offset,
spart, silo, siup, soffset, index_map, dir_map);
```

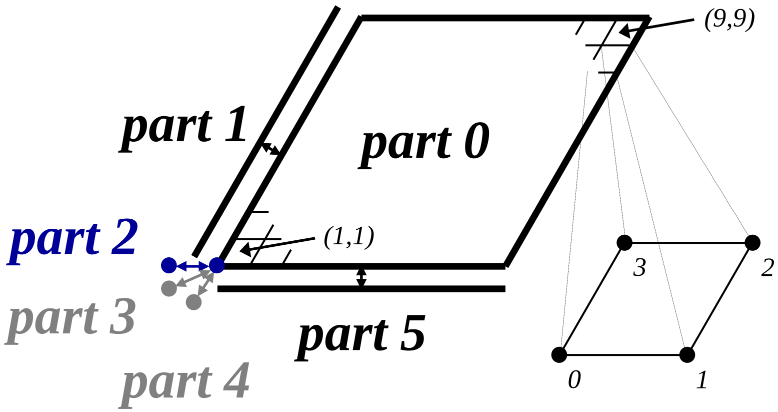
FEM example: Setting up the grid on process 0



Set shared variables for parts 0 and 5

```
int part = 0, spart = 5;
int ilo[2]  = {1,1}, iup[2]  = {9,1}, offset[2]  = {0,-1};
int silo[2] = {1,1}, siup[2] = {1,9}, soffset[2] = {-1,0};
int index_map[2] = {1,0}, index_dir[2] = {1,-1};

HYPRE_SStructGridSetSharedPart(grid, part, ilo, iup, offset,
spart, silo, siup, soffset, index_map, dir_map);
```

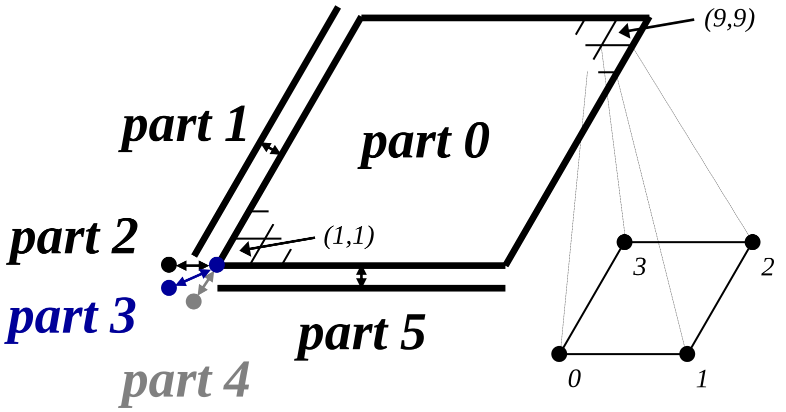
FEM example: Setting up the grid on process 0



Set shared variables for parts 0 and 2

```
int part = 0, spart = 2;
int ilo[2]  = {1,1}, iup[2]  = {1,1}, offset[2]  = {-1,-1};
int silo[2] = {1,1}, siup[2] = {1,1}, soffset[2] = {-1,-1};
int index_map[2] = {0,1}, index_dir[2] = {-1,-1};

HYPRE_SStructGridSetSharedPart(grid, part, ilo, iup, offset,
spart, silo, siup, soffset, index_map, dir_map);
```

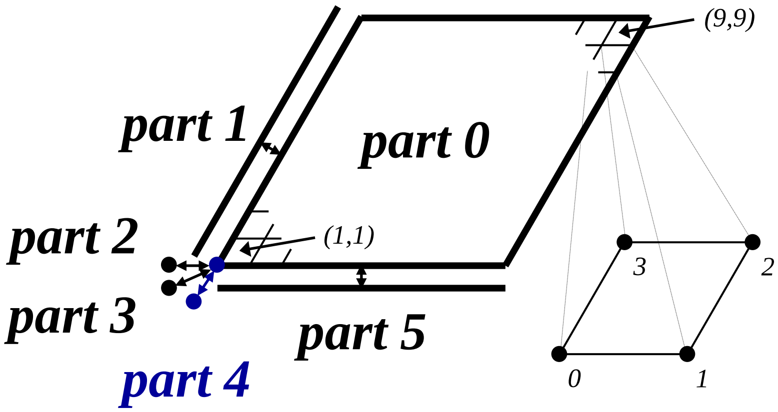
FEM example: Setting up the grid on process 0



Set shared variables for parts 0 and 3

```
int part = 0, spart = 3;
int ilo[2]  = {1,1}, iup[2]  = {1,1}, offset[2]  = {-1,-1};
int silo[2] = {1,1}, siup[2] = {1,1}, soffset[2] = {-1,-1};
int index_map[2] = {0,1}, index_dir[2] = {-1,-1};

HYPRE_SStructGridSetSharedPart(grid, part, ilo, iup, offset,
spart, silo, siup, soffset, index_map, dir_map);
```

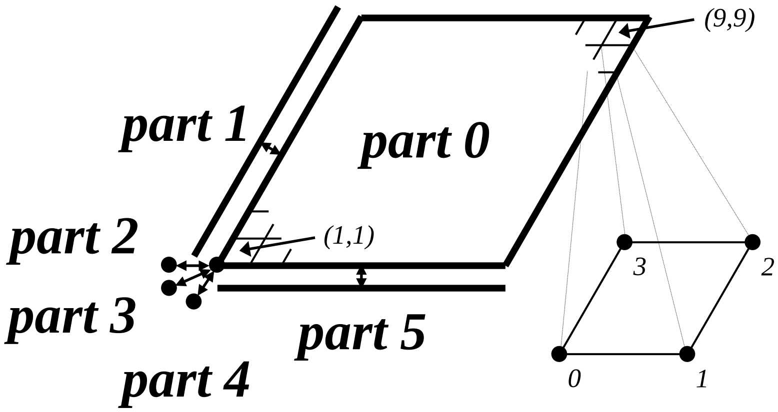
FEM example: Setting up the grid on process 0



Set shared variables for parts 0 and 4

```
int part = 0, spart = 4;
int ilo[2]  = {1,1}, iup[2]  = {1,1}, offset[2]  = {-1,-1};
int silo[2] = {1,1}, siup[2] = {1,1}, soffset[2] = {-1,-1};
int index_map[2] = {0,1}, index_dir[2] = {-1,-1};

HYPRE_SStructGridSetSharedPart(grid, part, ilo, iup, offset,
spart, silo, siup, soffset, index_map, dir_map);
```

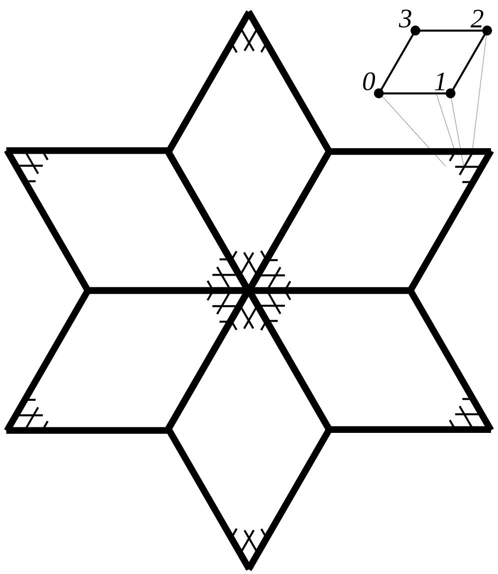
FEM example: Setting up the grid on process 0



Assemble the grid

```
HYPRE_SStructGridAssemble(grid);
```

FEM example: Setting up the graph on process 0

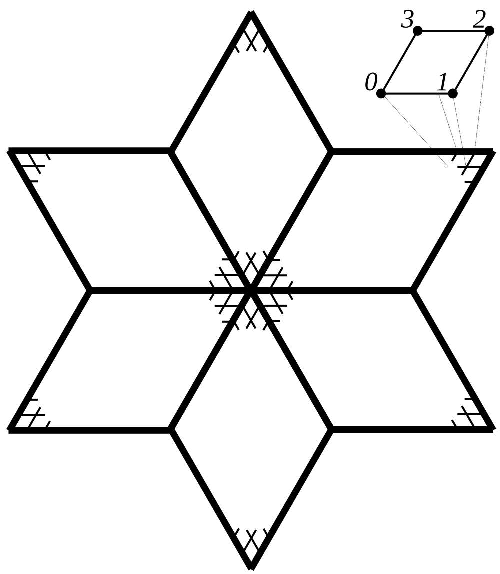


+ FEM

Create the graph object

```
HYPRE_SStructGraph graph;

HYPRE_SStructGraphCreate(MPI_COMM_WORLD, grid, &graph);
```

FEM example: Setting up the graph on process 0



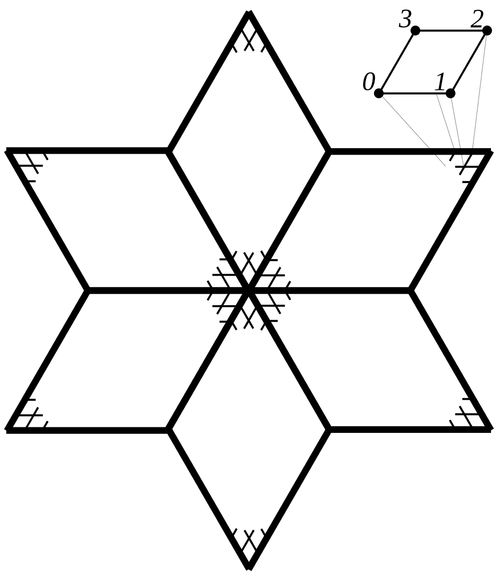
+ FEM

Set FEM instead of stencils for each part

(Set nonzero pattern of local stiffness matrix)

```
int part;

HYPRE_SStructGraphSetFEM(graph, part);

/* Optional: HYPRE_SStructGraphSetFEMSparsity() */
```

FEM example: Setting up the graph on process 0



+ FEM

Assemble the graph

```
/* No need to add non-stencil entries
 * with HYPRE_SStructGraphAddEntries() */

HYPRE_SStructGraphAssemble(graph);
```

FEM example: Setting up the matrix and vector

- Matrix and vector values are set one element at a time

- For matrices, pass in local stiffness matrix values

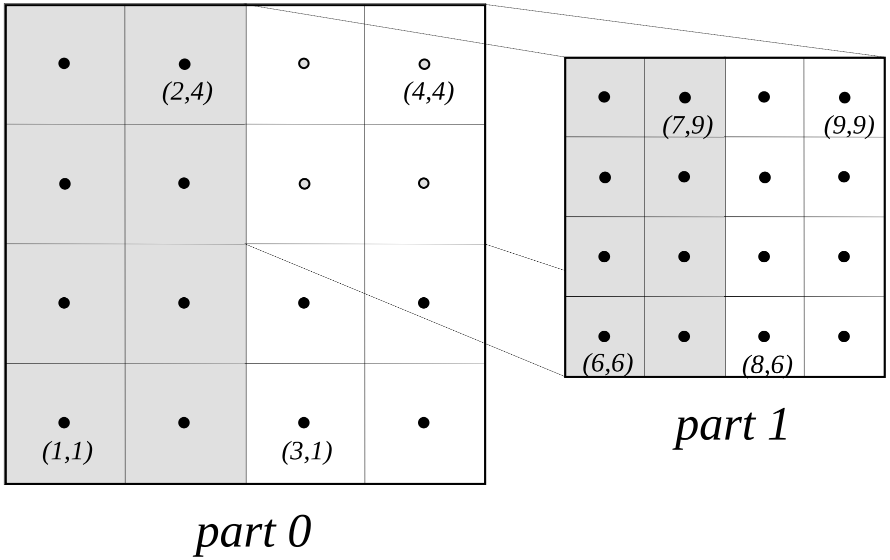
```
int part = 0;
int index[2] = {i,j};
double values[16] = {…};

HYPRE_SStructMatrixAddFEMValues(A, part, index, values);
```

- For vectors, pass in local variable values

```
double values[4] = {…};

HYPRE_SStructVectorAddFEMValues(v, part, index, values);
```

Structured AMR example (`SStruct`)

- Consider a simple cell-centered discretization of the Laplacian on the following structured AMR grid



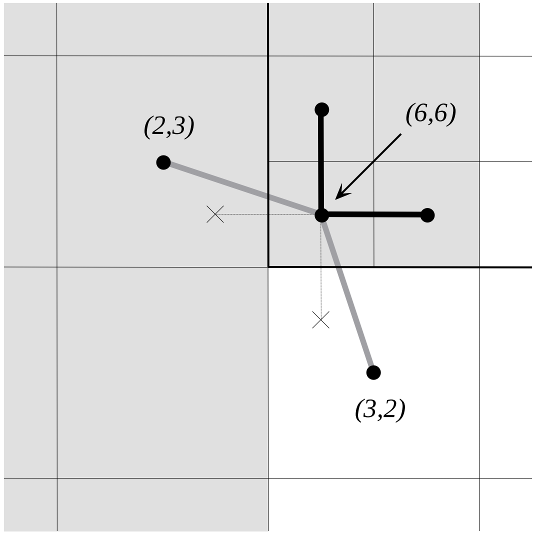
- Each AMR grid level is defined as a separate part
- Assume 2 processes with shaded regions on process 0 and unshaded regions on process 1

Structured AMR example (`SStruct`)

- The grid is constructed using straightforward calls to the routines `HYPRE_SStructGridSetExtents()` and `HYPRE_SStructGridSetVariables()` as in the previous block-structured grid example

- The graph is constructed from a cell-centered stencil plus additional *non-stencil entries* at coarse-fine interfaces

- These non-stencil entries are set one variable at a time using `HYPRE_SStructGraphAddEntries()`



Building different matrix/vector storage formats with the `SStruct` interface

- Efficient preconditioners often require specific matrix/vector storage schemes

- Between `Create()` and `Initialize()`, call:

```
HYPRE_SStructMatrixSetObjectType(A, HYPRE_PARCSR);
```

- After `Assemble()`, call:

```
HYPRE_SStructMatrixGetObject(A, &parcsr_A);
```

- Now, use the `ParCSR` matrix with compatible solvers such as BoomerAMG (algebraic multigrid)

Current solver / preconditioner availability via *hypre*'s linear system interfaces

Data Layouts	Solvers	System Interfaces: Struct	SStruct	FEI	IJ
Structured	Jacobi	✓	✓		
	SMG	✓	✓		
	PFMG	✓	✓		
Semi-structured	Split		✓		
	SysPFMG		✓		
	FAC		✓		
	Maxwell		✓		
Sparse matrix	AMS, ADS		✓	✓	✓
	BoomerAMG		✓	✓	✓
	MLI		✓	✓	✓
	ParaSails		✓	✓	✓
	Euclid		✓	✓	✓
	PILUT		✓	✓	✓
Matrix free	PCG	✓	✓	✓	✓
	GMRES	✓	✓	✓	✓
	BiCGSTAB	✓	✓	✓	✓
	Hybrid	✓	✓	✓	✓

Setup and use of solvers is largely the same (*see Reference Manual for details*)

- Create the solver
  ```
  HYPRE_SolverCreate(MPI_COMM_WORLD, &solver);
  ```
- Set parameters
  ```
  HYPRE_SolverSetTol(solver, 1.0e-06);
  ```
- Prepare to solve the system
  ```
  HYPRE_SolverSetup(solver, A, b, x);
  ```
- Solve the system
  ```
  HYPRE_SolverSolve(solver, A, b, x);
  ```
- Get solution info out via system interface
  ```
  HYPRE_StructVectorGetValues(struct_x, index,
  values);
  ```
- Destroy the solver
  ```
  HYPRE_SolverDestroy(solver);
  ```

Solver example: SMG-PCG

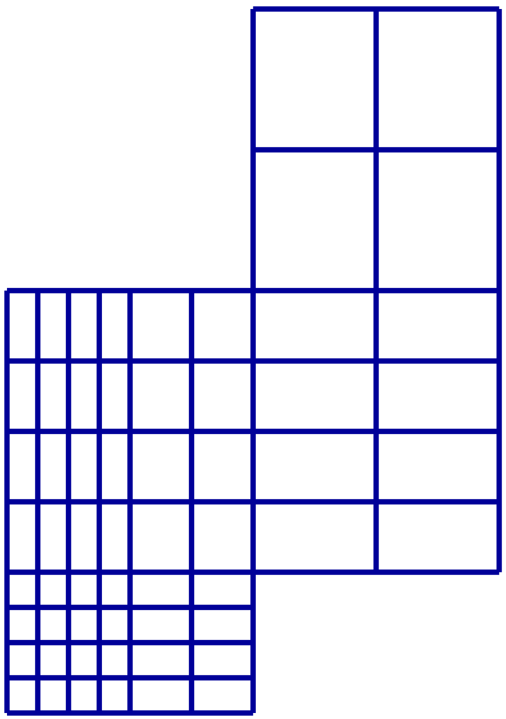
```
/* define preconditioner (one symmetric V(1,1)-cycle) */
HYPRE_StructSMGCreate(MPI_COMM_WORLD, &precond);
HYPRE_StructSMGSetMaxIter(precond, 1);
HYPRE_StructSMGSetTol(precond, 0.0);
HYPRE_StructSMGSetZeroGuess(precond);
HYPRE_StructSMGSetNumPreRelax(precond, 1);
HYPRE_StructSMGSetNumPostRelax(precond, 1);

HYPRE_StructPCGCreate(MPI_COMM_WORLD, &solver);
HYPRE_StructPCGSetTol(solver, 1.0e-06);

/* set preconditioner */
HYPRE_StructPCGSetPrecond(solver,
   HYPRE_StructSMGSolve, HYPRE_StructSMGSetup, precond);

HYPRE_StructPCGSetup(solver, A, b, x);
HYPRE_StructPCGSolve(solver, A, b, x);
```

SMG and PFMG are semicoarsening multigrid methods for structured grids

- Interface: `Struct, SStruct`
- Matrix Class: `Struct`

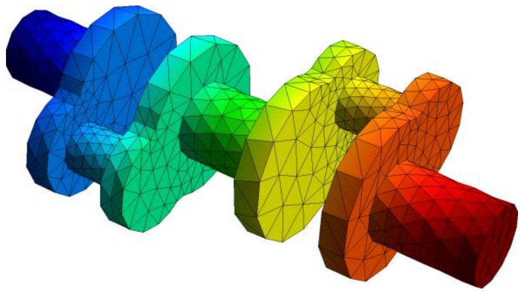
- SMG uses plane smoothing in 3D, where each plane “solve” is effected by one 2D V-cycle
- SMG is very robust
- PFMG uses simple pointwise smoothing, and is less robust

- Constant-coefficient versions!

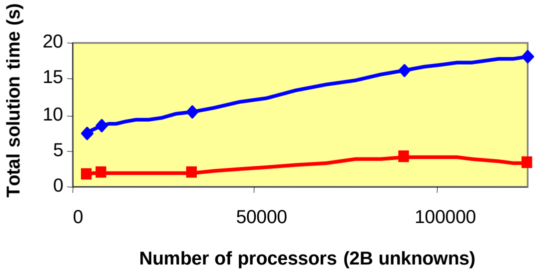


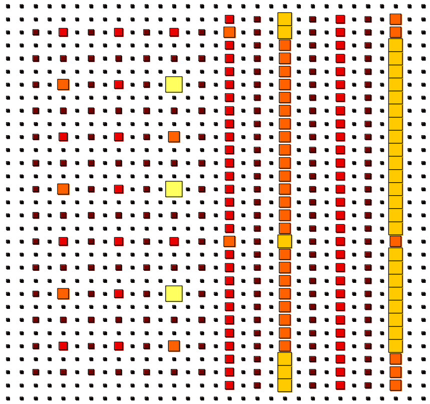
BoomerAMG is an algebraic multigrid method for unstructured grids

- Interface: `SStruct, FEI, IJ`
- Matrix Class: `ParCSR`



- Originally developed as a general matrix method (i.e., assumes given only A, x, and b)
- Various coarsening, interpolation and relaxation schemes
- Automatically coarsens "grids"
- Can solve systems of PDEs if additional information is provided



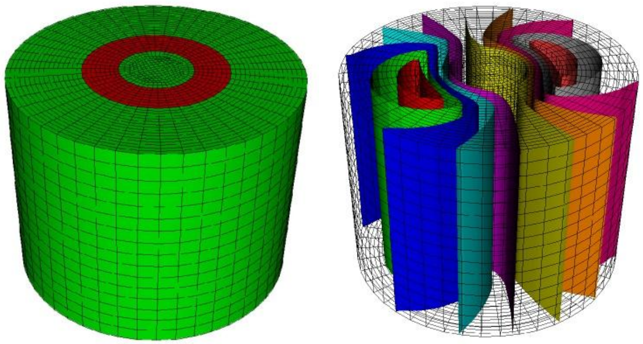


AMS is an auxiliary space Maxwell solver for unstructured grids

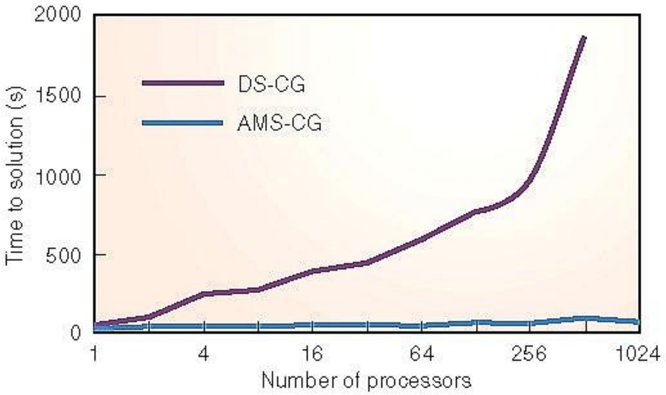
- Interface: `SStruct`, `FEI`, `IJ`
- Matrix Class: `ParCSR`
- Solves definite problems:

$$\nabla \times \alpha \nabla \times E + \beta E = f, \;\; \alpha > 0, \beta \geq 0$$

- Requires additional gradient matrix and mesh coordinates
- Variational form of Hiptmair-Xu
- Employs BoomerAMG
- Only for FE discretizations
- ADS is a related solver for FE grad-div problems.



Copper wire in air, conductivity jump of 10^6

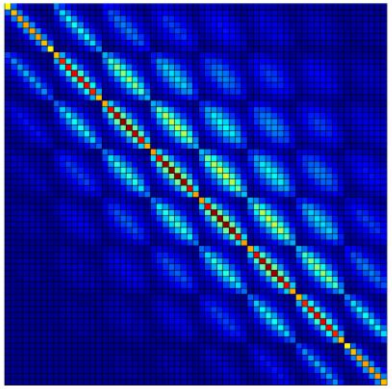


25x faster on 80M unknowns

ParaSAILS is an approximate inverse method for sparse linear systems

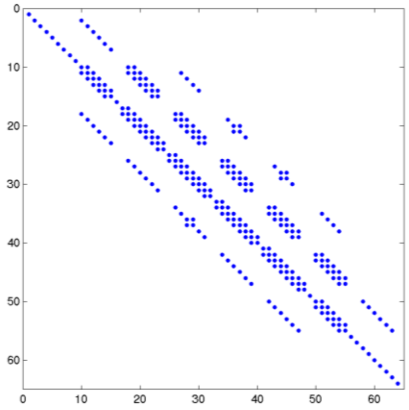
- Interface: `SStruct`, `FEI`, `IJ`
- Matrix Class: `ParCSR`

Exact inverse



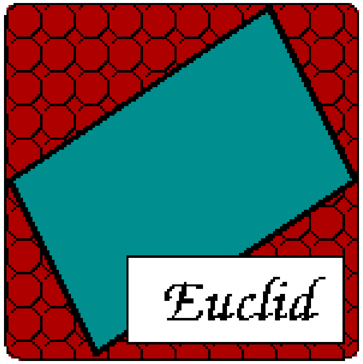
- Approximates the inverse of A by a sparse matrix M by minimizing the Frobenius norm of $I - AM$
- Uses graph theory to predict good sparsity patterns for M

Approx inverse



Euclid is a family of Incomplete LU methods for sparse linear systems

- Interface: `SStruct`, `FEI`, `IJ`
- Matrix Class: `ParCSR`



- Obtains scalable parallelism via local and global reorderings
- Good for unstructured problems

http://www.cs.odu.edu/~pothen/Software/Euclid

Getting the code

- To get the code, go to

  http://www.llnl.gov/CASC/hypre/

- User's / Reference Manuals can be downloaded directly

- A short form must be filled out (just for our own records)

Building the library

- Usually, *hypre* can be built by typing `configure` followed by `make`

- Configure supports several options (for usage information, type '`configure --help`'):

  '`configure --enable-debug`' - turn on debugging
  '`configure --with-openmp`' - use openmp
  '`configure --disable-fortran`' - disable Fortran tests
  '`configure --with-CFLAGS=…`' - set compiler flags

- Release now includes example programs!

Calling *hypre* from Fortran

- C code:

```
HYPRE_IJMatrix A;
int             nvalues, row, *cols;
double         *values;

HYPRE_IJMatrixSetValues(A, nvalues, row, cols, values);
```

- Corresponding Fortran code:

```
integer*8        A
integer          nvalues, row, cols(MAX_NVALUES)
double precision values(MAX_NVALUES)

call HYPRE_IJMatrixSetValues(A, nvalues, row, cols, values)
```

Reporting bugs, requesting features, general usage questions

- Send email to:

  hypre-support@llnl.gov

- We use a tool called Roundup to automatically tag and track issues

Thank You!

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